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MANUAL

OF

ARTILLERY SURVEY.

PART I.

1924.

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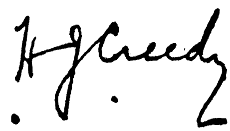

THE WAR OFFICE,

April, 1924.

MANUAL OF ARTILLERY SURVEY.

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CHAPTER I.

1. Introduction.

Survey may be defined as the determination of the relative positions of terrestrial objects. Its application to artillery work is threefold, *i.e.*:—

(*a*) It supplies the data from which are prepared the maps indispensable to any military operations.

(*b*) It enables the correct relative positions of a gun and its target to be determined so that, when the target cannot be seen from the gun position or the battery O.P., the line and map range can be determined before fire is opened.

(*c*) It enables the gun to be laid accurately on any desired line without the necessity of ranging or registration.

2. The preparation of maps is the duty of the R.E. Field Survey services. The survey operations required for (*b*) and (*c*) are performed by the Royal Artillery, either by special units or by trained personnel in the brigades and batteries.

3. These operations fall into two separate categories :—

(i) Determination of the correct positions of our own guns, and of their targets, on the earth's surface. From these determinations their *relative* positions, *e.g.*, the line and range, can afterwards be deduced.

(ii) The laying of the gun correctly on any given (predetermined) line.

4. **The location of targets** which cannot be visited by a surveyor, requires, as a rule, special methods of survey which would be unnecessarily complicated if applied to the location of our own guns. The location of hostile batteries particularly has led to the evolution of methods, known as "flash spotting" and "sound ranging," so specialized that they can only be effectively operated by complete and separate units (Survey Companies, R.A.) trained and drilled in their use.

These special methods are dealt with in the manuals of Flash Spotting and Sound Ranging, and are outside the scope of this book.

5. In this manual, descriptions are given of the principal survey methods and processes likely to be required either by the Survey Companies, R.A., as a basis for their special work, or by the personnel of brigades and batteries for directing the fire of their guns.

6. For fuller descriptions of these and other survey methods, such as will usually be employed only by the R.E. Field Survey Services (and therefore only by the R.A. in the absence of such units), reference should be made to the Text-book of Topographical Surveying, and to the Manual of Map Reading and Field Sketching.

2. Principles of Survey.

1. Determination of the relative positions of two or more points implies measurement of the linear distances between them, and of the angles made by the line joining any two with the lines joining these two to other points.

It is convenient to measure and record the horizontal rather than the actual distances. The relative positions of the two points are expressed in terms of the horizontal distance between them and the difference in their heights, measured from some common horizontal datum, usually mean sea level.

2. In making such measurements it has to be recognized from the outset that whatever instruments are used, and however great the care exercised, it is impossible to make them with absolute accuracy. Each measurement will be burdened with an error, great or small, according to the manner in which it is made; and since the survey of a number of points implies a succession of measurements, each starting from the point at which another terminated, the errors in relative position due to inaccurate measurement are cumulative; the final error of position at the termination of any measurement will be the error in that measurement superimposed on the error at the starting point. Over a long series of measurements the final error will be the algebraic sum of all the errors in each measurement.

3. It is a fundamental principle of accurate survey that the work to be done should be regarded from the first as a whole, and that when the maximum permissible error in any part of it has been decided the relation between the degree of precision of each measurement and the number of successive measurements in any series should be such that the accumulated error at any point in no case exceeds the permissible maximum.

4. A second fundamental principle is consequently that the work should be so arranged that it contains in itself a number of internal checks to enable the surveyor to gauge both the precision of his work and the amount of error accumulated in any given series.

5. The errors generated in any survey are of two kinds: errors of length and errors of direction. These are essentially different in character, and must be gauged and controlled by checks of different kinds; the nature of these checks will be better understood when explained in relation to any particular method of work.

6. The precision of any work may be gauged in various ways; in part as a result of the checks in the work itself, and in part from the individual observations or measurements. Thus, if the same measurement is repeated in the same manner two or more times, the accordance between independently effected measurements of the same quantity sometimes gives in itself a certain criterion of their accuracy. Again, though it is impossible to say what may be the absolute error in the observed distances and directions between any two points, if a series of measurements is continued from point to point in such a way as to terminate at the point from which it originated, the total error generated in all the measurements can be exactly determined, and the degree of accuracy of any one of them inferred at least approximately.

7. In this manner it is possible to deduce from experience the errors which may be expected from the use of certain types of instruments and certain methods of work. It is, moreover, possible, by determining in each case the total error generated at the end of a series of measurements, to " distribute " it systematically back throughout the series, by correcting each individual measurement so as to reduce to zero the algebraic sum of the errors in the corrected measurements. This is termed " closing " a series, and, though it will not eliminate the errors altogether, it will materially reduce their magnitude, and entirely prevent their accumulation beyond a certain point.

8. It may not be, and generally is not, convenient in all cases to terminate every series of measurements at its starting point. Nor in practice is it necessary to do so. The usual practice is to fix the positions of a certain number of points in any area by instruments and methods whose precision is so great that the errors in them are inappreciable in comparison with the errors generated in subsequent series of measurements, starting at one, and terminating at another of them, and executed in a less but sufficiently accurate manner. For example, if the distance between two points A and B has been determined in such a way that there is a reasonable certainty that the error in the determination does not exceed 1 foot, and a series of measurements starting at A and ending at B shows a terminal discrepancy in the position of B amounting to 50 feet, the ratio of the error in the original determination of the length of AB to the error generated in the subsequent series is so small that the corrections which have to be made to this second series are practically identical with those which would have been applied had the length of AB been known with absolute exactness. In other words, for purposes of the second series the position of the two points A and B may be regarded as being without error.

9. A third principle of survey, then, is that every series of measurements should start from one point and close on another whose position has been previously determined by a more accurate series. In this way it is possible to prevent the accumulation of error beyond a certain magnitude in any part of an extended survey, without employing the most accurate, and, therefore, the slowest methods in all parts.

10. As a general rule, the more accurate the method the slower it is in execution. Economy of time and effort without loss of accuracy is secured by proper selection of the relation between the method employed and the distance over which it has to be carried. In the first instance a limited number of points are fixed by a method of the first order of accuracy. Between these, other and more numerous points are fixed by a quicker and less accurate method, and these points in their turn serve as a basis for other methods in a diminishing ratio of accuracy and an increasing ratio of speed until all the points have been dealt with and the work is complete.

11. These principles have their fullest applications in the complete systems of survey required for mapping. Their exact nature will be more fully understood when the actual methods of survey have been described. They are, however, applicable in some degree to survey operations of all kinds, and should never be lost sight of, or neglected.

12. Experience shows that surveys originally intended to be limited in scope and extent, very frequently have to be extended ultimately far beyond their original boundaries, or have to be joined to, or incorporated in other surveys. If these principles have been applied from the outset, the adjustment of one work to the other is greatly simplified, and the surveyor is able to say at any point the degree of reliance which may be placed on his work, and to estimate the errors which may be expected in any given extension of it.

3.—Survey Methods.

1. The most obvious way of measuring the distance apart of two points is direct linear measurement along the ground with a graduated tape or chain. Since, however, it is impossible to handle in the field tapes of more than comparatively short length, direct linear measurements, for determination of distances exceeding a few hundred yards, involve many successive measurements, and are, in consequence, liable to considerable accumulations of error.

2. Moreover, it must be remembered that the surface of the ground will often present obstacles to the passage of the surveyor, which will take time to surmount, and may seriously increase the difficulty and inaccuracy of his measurements.

3. So much is this the case, that direct linear measurement is rarely used, except when it is absolutely unavoidable ; it is, whenever possible, replaced by calculations depending on angular measurements, by the process known as Triangulation.

4. If three points are selected so as to form a triangle on the ground, and the three angles of this triangle are measured, the lengths of all three sides can be calculated if any one of them is known.

5. The process of triangulation consists in the selection of a number of points arranged so as to form a series or network of contiguous triangles. Each point forms the vertex of one or more triangles, and at each the angles between the lines joining it to the other points surrounding it are measured with a suitable instrument. Two of the points are chosen so that the ground between them is open, level, and especially suitable for direct linear measurement, and the distance between these two points is measured with special care.

This measurement forms the " base " from which the other sides of one or more triangles are calculated. These sides, in their turn, serve as bases for the solution of other triangles, and so on, until the distances between all the points have been determined.

6. These calculated values can be checked at intervals throughout the work by direct measurement between any two points conveniently situated for the purpose.

7. Angular as opposed to linear measurement has the great advantage that the instruments with which it is effected do not require corrections for temperature, tension, &c. ; it does not require a large number of consecutive observations to effect a single measurement ; it is not affected by obstacles to the passage of the surveyor, and lastly it lends itself rather more readily to the provision of the internal checks in the work referred to in Sec. 2.

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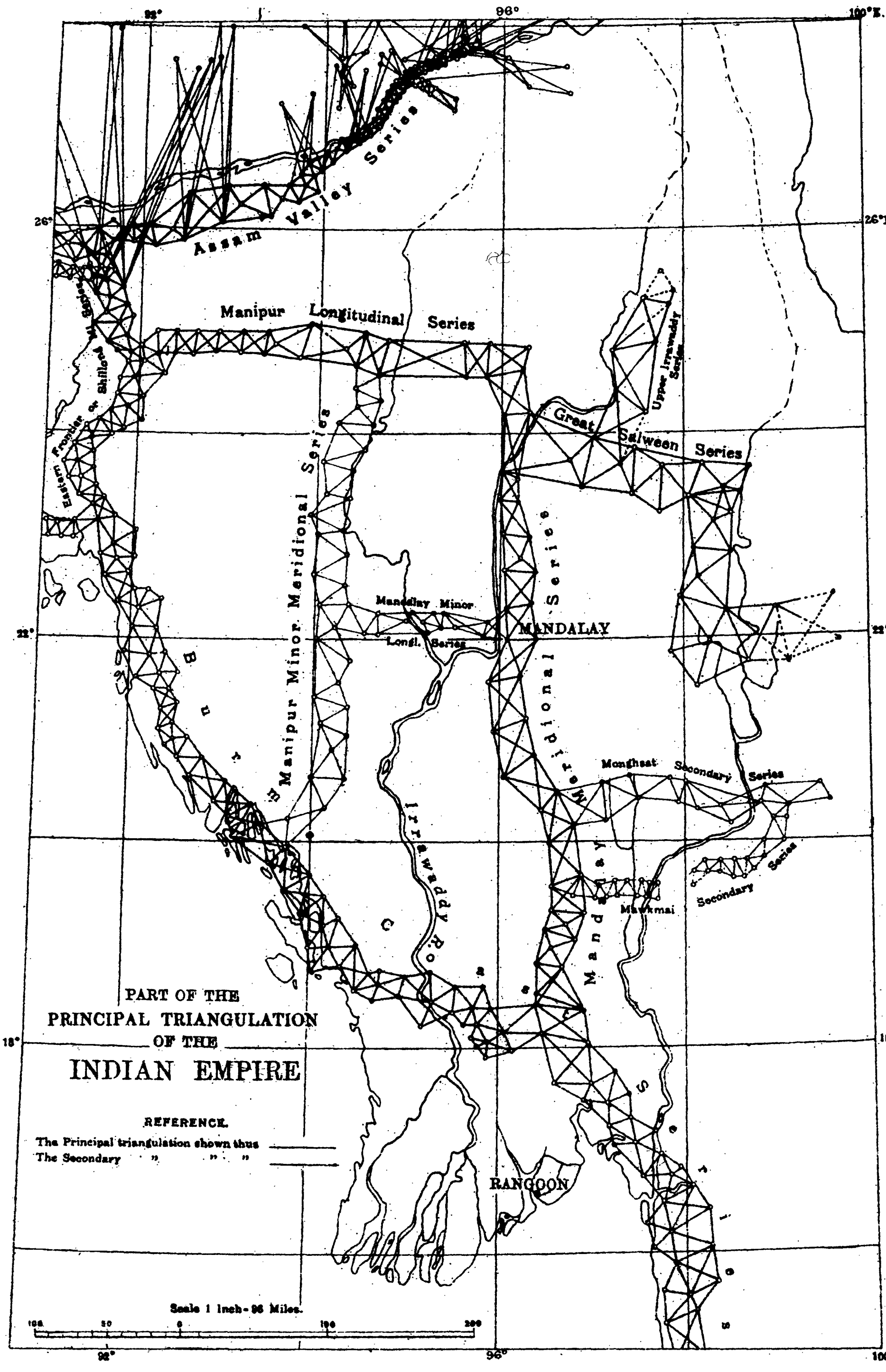
Fig.1



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8. Triangulation, whenever it can be done, is always far quicker and more accurate for a given expenditure of time and labour than any system of direct linear measurement, and forms the basis of practically all modern surveys. Even when the nature of the country, *e.g.*, large flat forests, or swamps covered with thick vegetation, renders it so slow, difficult, and costly that it has to be to a great extent replaced by other methods, it is often found necessary to carry out a bare minimum of triangulation, even at great expense, to control whatever alternative method is used for the bulk of the work.

9. Triangulation is invariably graded in accuracy, and is classified as :—

(*a*) Primary triangulation (sometimes called principal or geodetic triangulation).
(*b*) Secondary triangulation.
(*c*) Minor triangulation.
(*d*) Tertiary triangulation.

Primary or geodetic triangulation forms the basis of all large first-class surveys.

All subsequent triangulation, secondary, tertiary, &c., is based upon, and adjusted to the primary work.

In secondary triangulation the accumulation of error is confined to the distance which separates the points of the primary triangulation.

Secondary points, in their turn, limit the accumulation of error in the less accurate work, and so on.

In the primary triangulation alone there is nothing to arrest the accumulation of error. The observations for it must, consequently, be sufficiently accurate in themselves to avoid, as far as possible, any accumulation of error of embarrassing extent. Primary triangulation is, therefore, always executed with the maximum possible care and refinement.

10. Primary triangulation is both slow and expensive. It is not, therefore, as a rule, carried out as a network over the whole area to be surveyed (though this was done in some early surveys, such as the Ordnance Survey of Great Britain). In modern work it is usually arranged in the form of a number of widely separated series or chains of triangles forming a " gridiron " over the area, as shown in Fig. 1.

The intervals between these series are then bridged with other series of " secondary " work, and finally the remaining intervals are covered with a complete network of minor and tertiary work until a sufficient number of points for the detail survey has been fixed.

11. When triangulation is impossible, the framework of points for the detail surveyor is provided by "traversing," a combination of angular with linear measurements. A traverse consists of a connected series of straight lines on the earth's surface of which the lengths and bearings have been determined. Like triangulation, traverses should always be graded in accuracy. In the most accurate forms the included angles between any two consecutive lines are measured with the same instruments as are used for primary and secondary triangulation, and in a generally similar manner. Linear measurements are generally effected with invar tapes (*see* Chap. V, Sect. **18**). In less accurate

work angles are measured with a smaller instrument and linear distances with a steel tape, chain, by subtense bar (Chap. VII, Sect. 26). by perambulator (a wheel with "cyclometer" attachment), or even by pacing.

12. Topographical traverses may be classified under two main categories, viz :—

(*a*) Theodolite traverse,
(*b*) Compass traverse,

according to the nature of the instrument used for measuring the angles.

Traversing can, however, be done with the plane table, or by the aid of a sextant or any other instrument for measuring angles. Theodolite traversing is largely used for town plans and large scale surveys generally, and for the framework of points in very flat, highly cultivated, or densely wooded countries.

13. The framework of points of practically all surveys has hitherto been fixed either by triangulation or by traverse. In the future, it is possible that aerial photography may also be used for this purpose.

14. The detail survey is based on this framework and may be done either by plane-table (Chap. VIII), by aerial photography, by traverse, or by a combination of all these.

15. Plane-tabling is essentially nothing more than triangulation executed graphically, and is, in reasonably open country, the most rapid and accurate method of detail survey.

It is, however, unsuitable for flat, enclosed country, where the view is restricted. In such country aerial photography is probably the most satisfactory method, although for accurate work aerial photography requires a considerably greater number of control, or framework, points than plane-tabling.

16. Aerial photography has the characteristic, not possessed by any other method, that it is not absolutely necessary for a surveyor to go over the ground surveyed. It is therefore likely to be of especial value in war for surveys of ground occupied by the enemy.

17. The actual measurements constituting any survey are done by one or other of the above methods. In addition to this, however, it is necessary, in the survey of any country, to locate the whole survey correctly in position on the surface of the earth. This is done by astronomical observation. By means of the stars it is possible to determine the latitude of any point, and, when this is known, to determine time at that point. From the determination of local time the longitude of the point can be found by means of the electric telegraph (or by wireless).

The stars and other heavenly bodies also enable the direction of the true north to be determined, and, therefore, afford a useful means of checking the *directions* of both triangulation and traverses.

18. Astronomical observation has been used in rapid reconnaissance work (*e.g.*, certain boundary commissions) in place of, or to supplement, triangulation or traversing.

Astronomical determinations of position, *i.e.*, latitude and longitude, unless done with very refined instruments, are not sufficiently accurate for topographical purposes. Each observation is, however, entirely independent of others. Errors are not, therefore, cumulative. Astronomical determinations of position, even though subject to considerable errors, can be, and are used to control the *accumulation* of errors in an extended survey.

Astronomical determinations of *direction*, however, can be made very accurately with ordinary field instruments. It is for this purpose that they are chiefly useful to the artillery surveyor.

CHAPTER II.

MAP PROJECTIONS AND GRID SYSTEMS.

4. General Principles.

1. The survey processes and methods described in the preceding chapters imply a succession of measurements of one sort or another from point to point. It will be noted that it is a succession of measurements, and that the distances and mutual directions of *all* the points from each other are not measured directly. It is not difficult to see that this succession of measurements, between what may be called adjoining points, furnishes data from which the relative positions of any points, however widely separated, may be calculated.

2. Since, however, the points are all situated on the curved surface of the earth, the calculation necessary, when the survey extends over any considerable area, must take the figure of the earth into consideration.

For example, in triangulation, although each triangle, for purposes of calculation of the length of its sides, is treated as a plane triangle, it is not correct to regard all the triangles as lying in the same plane.

3. It is impossible, therefore, to stop any survey work at the stage when only the distances apart and mutual directions of adjoining points on the system have been determined. It is necessary to define the position of all points with reference to an origin or datum common to all of them ; in short, to calculate, at the conclusion of the outdoor work, the co-ordinates of each point with respect to two axes.

4. The surface of the earth, on which the points lie, is a slightly irregular spheroid whose polar diameter is about 26 miles shorter than the equatorial.

For the purpose of the present description the earth may be considered as a sphere, because the irregularities are small compared with its size.

5. A sphere is a symmetrical figure such that any point on its surface is exactly like any other point. To define the positions of any particular points with respect to one another some lines of reference must be selected.

The earth is a rotating sphere, and its axis of rotation is a definite line differing from every other diameter. The ends of this diameter are called the poles.

The equator is the line of intersection, with the surface of the sphere, of a plane which bisects this diameter and is at right angles to it.

The equator is, therefore, a definite and unique line upon the earth's surface.

All circles on the earth's surface which divide it into two equal parts are called "great circles." The equator is, therefore, a great circle.

Great circles which pass through the poles are called meridians, and all meridians cut the equator at right angles.

Any particular meridian and the equator, therefore, furnish two lines cutting each other at right angles, which can conveniently be used as axes of a system of co-ordinates.

Such systems are usually known as "geographical," and, in British practice, the axes are always the equator and the meridian of Greenwich.

The position of a point on such a system is defined by its latitude and its longitude expressed in each case as an *angle*.

The latitude represents the angle, subtended at the centre of the sphere, by the arc of the meridian through the point, lying between the point and the equator. The longitude represents the angle between the planes of the meridian through the point and the meridian of Greenwich.

These two angles completely define the position of any point.

From the latitudes and longitudes of two points their distances apart and mutual directions can be calculated, and conversely if the latitude and longitude of one point is known and the distance and direction of another point from it can be measured, the latitude and longitude of the second point can be calculated.

6. This system is generally employed for surveys extending over a large area such as the survey of India. The positions of all triangulated points are defined by their latitudes and longitudes, to two or three decimal places of a second.

From this data it would obviously be a simple matter to plot the position of any point on a sphere representing the earth on the required scale. For convenience in plotting it would be desirable to rule, on such a sphere, a series of lines representing meridians and parallels at fixed and suitable intervals depending on the scale.

Thus, these lines might be ruled at intervals of 5 or 10 minutes, forming a sort of network over the whole surface of the sphere, and on this network the position of any points could easily be interpolated.

7. In preparing maps, however, it is not practicable to plot thus on the surface of a sphere. The ground, and all the points on it, have to be represented on a flat sheet of paper ; consequently it is a necessary preliminary to devise some means of representing the network of parallels and meridians, referred to in para. 6 and known as a "graticule," on a flat, in place of a curved, surface.

It is naturally impossible to represent *exactly* any considerable area in this way. The representation of lines on a curved surface upon a flat one can be done only approximately. Such approximate representations are known as "map projections."

A " map projection " is not necessarily a projection of the surface of the earth on to a plane surface in a geometrical sense. The majority of useful map projections are not geometrical projections at all.

5. Map Projections.

1. A " map projection " may be defined as any systematic way of representing meridians and parallels on a plane surface.

A great variety of such projections have been devised, each of which has certain characteristic properties, which render it more suited to one purpose than another.

2. (*a*) In appraising the merits of any projection the following are the characteristics which are generally considered :—

(i) The accuracy with which it represents length in any particular direction.
(ii) The accuracy with which it represents areas.
(iii) The accuracy with which it represents shape.
(iv) The ease with which it can be constructed.

(*b*) Correct representation of shape is confined to the shape of small areas and obviously implies :—

(i) Correct representation of angles.
(ii) That the scale at any point is the same in all directions (though not necessarily the same as in other parts of the projection).

3. The majority of useful map projections can be classified under three headings :—

(i) Zenithal.
(ii) Conical.
(iii) Conventional.

These names describe the method of construction. Each projection is usually given a second name defining its principal characteristic, and to this is sometimes added the name of its inventor.

Zenithal projections are mainly used for altas maps on very small scales. The projections used on large scale maps, which are those with which the artillery surveyor is mainly concerned, will almost always be in the second or third category.

4. A detailed description of the various map projections likely to be used in the field is beyond the scope of this book, nor is it necessary for the artillery surveyor to know how they are arrived at.

Whatever projection is used it is obvious that, since the length on the surface of the sphere represented by a given angular interval of longitude decreases continuously from the equator to the pole, a graticule represented on paper will usually appear as a network of quadrilateral figures of which the side representing the parallel nearest to the pole is shorter than that nearest to the equator.

To plot a point by its latitude and longitude in such a quadrilateral implies the preparation of separate scales for latitude (the north and south sides of the quadrilateral) and for each parallel. Moreover, for the parallels at least, separate scales must be prepared for each quadrilateral.

6. Rectangular Co-ordinates and Grid Systems.

1. For military purposes a simpler and quicker system is desirable, and this is obtained by superimposing on the graticule, after it is drawn out on the plane surface, a system of rectangular co-ordinates.

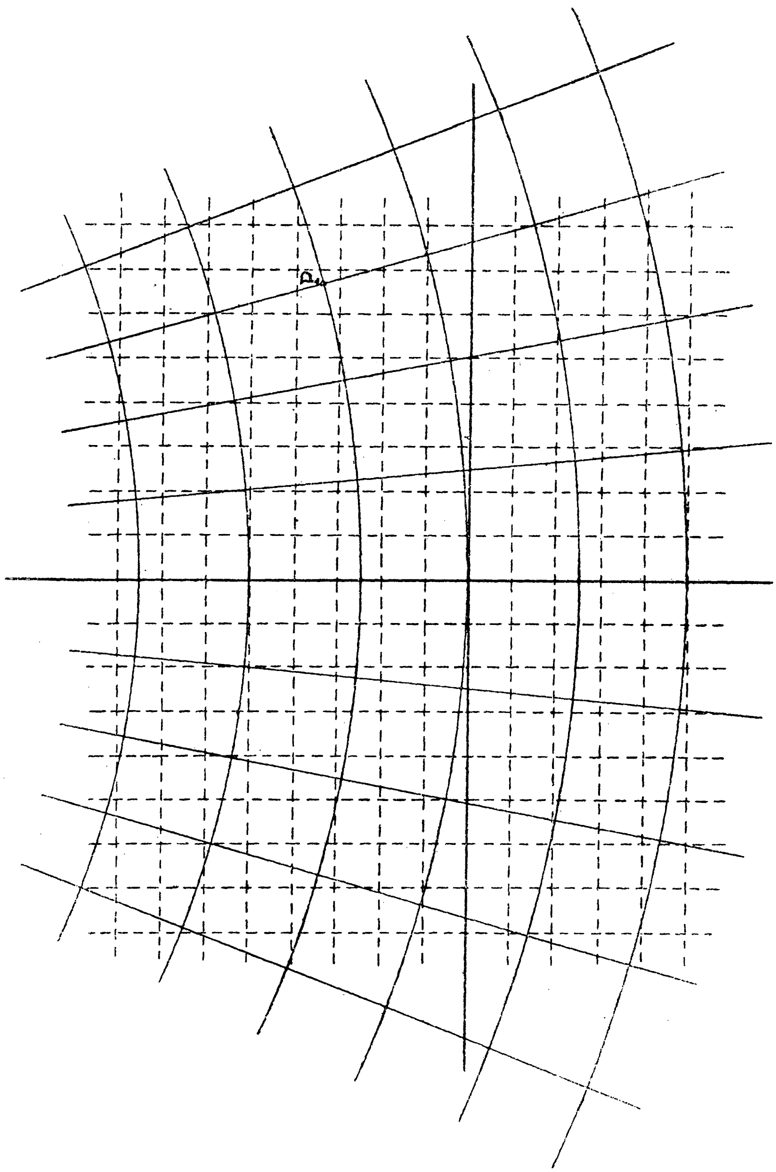

Fig. 2.

For example in Fig. 2,

If the graticule is represented by the continuous lines and the rectangular system by the dotted lines, it is easy to see that the rectangular co-ordinates of any point on the graticule, such as P, can be deduced either graphically, or by calculation, from the method of construction of the graticule and the relation of the axes of the rectangular system to it.

It can be seen further that the rectangular co-ordinates of *any* point whose latitude and longitude are known, *i.e.*, of any trigonometrical point, can be calculated in a similar way, and that once this calculation has been done it will be much quicker and simpler to plot the point on the rectangular system, for which only one scale is required, than by means of the graticule.

2. Once these calculations have been made for the points of the primary triangulation, all calculations referring to other points, which are interpolated between, and based on the primary work, can be done with quite sufficient accuracy on the rectangular system.

3. The artillery surveyor is not concerned with primary work, which is within the province of the R.E. He is therefore able to carry out his entire work on a rectangular system, in which the calculation of the mutual bearings (referred to the rectangular co-ordinates or grid system) and distances apart of points depends on very simple formulæ, whereas the formulæ connecting bearing and distance with longitude and latitude are rather complicated, and involve the use of special tables.

The advantages of a rectanglar system over any geographical or spherical system, for artillery purposes, is so overwhelming that there can be no doubt that the former will always be preferable for military purposes. It is, nevertheless, important that the artillery surveyor should understand that there is a definite relation between the rectangular grid system and the true graticule, and also in a general way what this relation is.

4. In reverting to Fig. 2, it will be noted that the north and south axis of the rectangular system is made to coincide with a particular meridian, and that the east and west rectangular axis touches a particular parallel at the point where the latter cuts this meridian.

This may be regarded as a normal relation between the rectangular system and the graticule. The particular meridian and parallel, which thus define the origin of the rectangular system, will usually be selected so as to fall somewhere near the centre of the area to be surveyed.

5. Two further points should be noted, one, that grid north and true north, as represented by the direction of the meridians on the graticule, only coincide along the meridian passing through the origin, and that the further the grid is carried to east or to west of this meridian the greater is the angle between the grid and true north.

This angle is known as the "convergence" of the meridians, and depends for any given meridian on the nature of the projection.

Formulæ from which it may be calculated in any particular case can be obtained from the R.E. Survey Services.

When squared or gridded maps are available, points on the graticule are usually shown in the margin. By scaling off the distance of such points at the top and bottom of a map sheet from the nearest grid lines, the convergence in any particular sheet or area can be calculated with sufficient accuracy for artillery purposes.

The second point to note is that, since on a rectangular system, co-ordinates measured east or north of the origin are conventionally regarded as positive and those to the south or west negative; for each point defined by certain figures there will be three other points whose co-ordinates are numerically the same, and distinguished from the first only by the signs, positive or negative, as the case may be, placed before the figures.

This is likely to be a cause of mistakes or confusion, and is generally obviated by adding algebraically to the co-ordinates of all points in the area an arbitrary large number, sufficiently large to make the co-ordinates of all points with which the surveyor is likely to be con cerned positive.

Thus in the squared 1/20,000 military maps of England the origin of the rectangular co-ordinates is a point in the Isle of Wight.

To all east and west co-ordinates the number 500,000, and to all north and south co-ordinates the number 100,000, has been added, these figures being sufficient to ensure that all co-ordinates in the United Kingdom will be positive.

This is, of course, equivalent to moving the *origin for purposes of numbering* to a point 500 000 metres west and 100,000 metres south of the true origin, as defined by the relation between the grid system and the graticule.

6. The divergence between grid north and true north on a rectangular system is not in practice an inconvenience.

The direction of true north in the field can only be determined by an astronomical observation and subsequent computation, or, for rough work, by compass, the variation of the compass from true north being known.

The additional computation required to determine grid north from an astronomical observation is very slight, while for compass work it is only necessary to know the compass variation from grid instead of true north.

7. Rectangular systems of co-ordinates cannot, however, be extended indefinitely in every direction. The graticule on many types of projection can be extended indefinitely in *one* direction without increase of distortion, (for example, the projection used in France by the French Army in 1918, the conical orthomorphic projection of Lambert, is capable of indefinite extension to east or west without increase of distortion) but it may be found that when the angle between grid and true north increases beyond a certain point the divergence may be a source of mistakes or confusion.

8. When, therefore, operations extend over a large area, it may be desirable to employ more than one system of rectangular co-ordinates, even though the projection is uniform and continuous over the whole area. For example, the system of *rectangular* co-ordinates used by the British Army of the Rhine in 1919 was based on a different origin and a different meridian to the system used in France and Belgium in 1918, although the projection was the same.

The rectangular system is, of course, continuous over the whole area in which it is employed, and, provided that the co-ordinates of any two points in the *same* system are known, their distances apart and the grid bearing of one from the other can easily be calculated.

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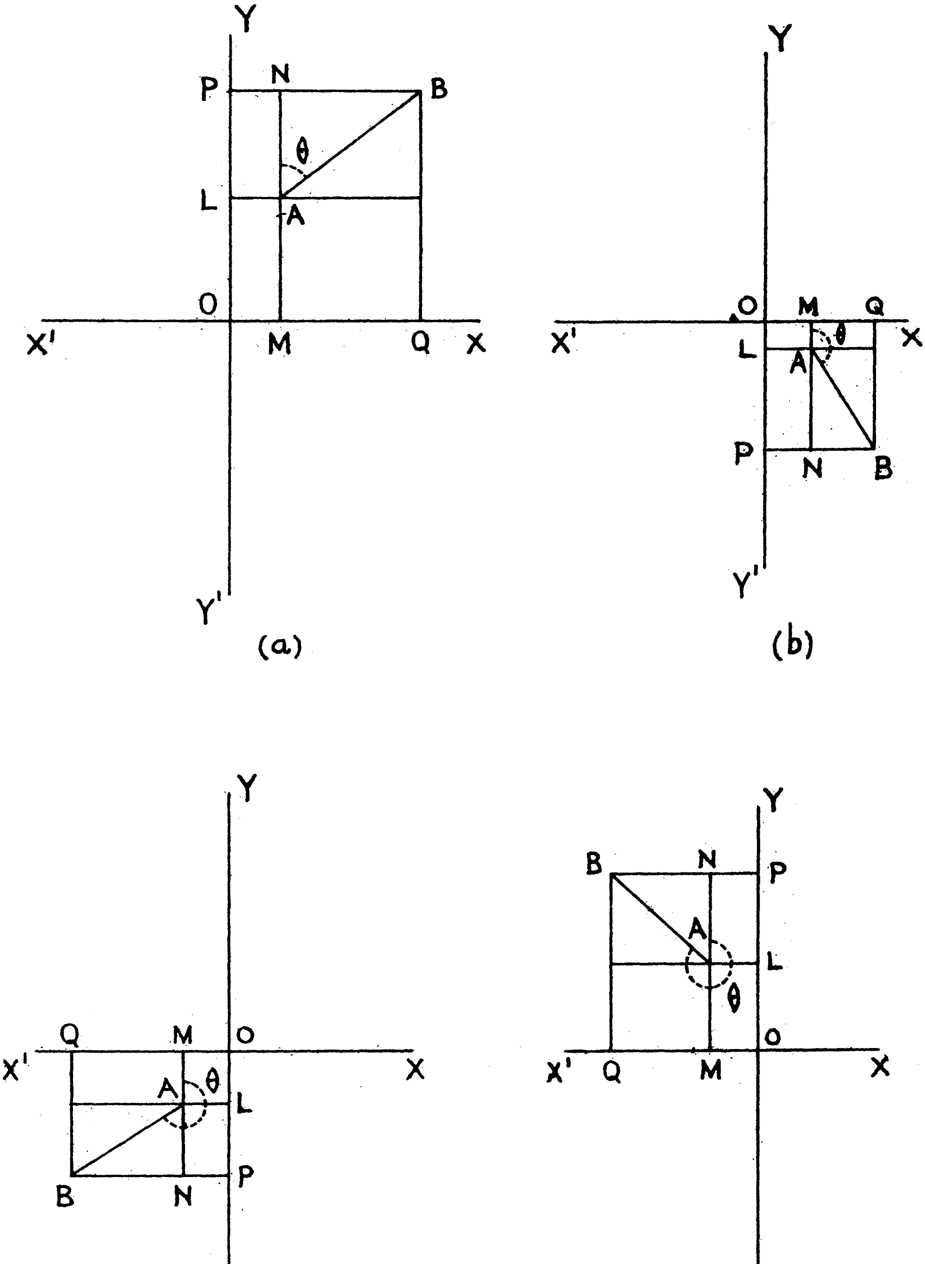

Fig. 3.

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7. Computation of Bearings and Distances.

1. In Fig. 3 let A and B be two such points and XOX′ and YOY′ the axes of the co-ordinate system.

Then BP, BQ are the co-ordinates of B, and AL, AM the co-ordinates of A.

The grid bearing from A to B is the angle NAB*, called θ.

$$\text{Hence } \tan\theta = \frac{BN}{AN} = \frac{BP - PN}{NM - AM} = \frac{BP - AL}{BQ - AM}$$

$$\text{(i) } \therefore \tan\text{, grid bearing} = \frac{\text{Difference of Easting co-ordinates of A and B}}{\text{Difference of Northing co-ordinates of A and B}}$$

$$\text{Again } \frac{BN}{AB} = \sin\theta \text{ and } \frac{AN}{AB} = \cos\theta$$

$$\therefore AB = \frac{BN}{\sin\theta} = \frac{BP - AL}{\sin\theta}$$

$$\text{(ii) } \therefore \text{Distance AB} = \frac{\text{Difference of Easting Co-ordinates of A and B}}{\text{sin grid bearing}}$$

$$\text{And AB} = \frac{AN}{\cos\theta} = \frac{BQ - AM}{\cos\theta}$$

$$\text{(iii) } \therefore \text{Distance AB} = \frac{\text{Difference of Northing co-ordinates of A and B}}{\text{cos grid bearing}}$$

Example—

Given the co-ordinates of A and B, to find the grid bearing AB and the length AB in metres.

	E	N	
Co-ordinates A	454750·3	164692·3	
Co-ordinates B	456183·6	162599·1	
Difference Easting =	1433·3	2093·2	= Difference Northing

Log difference Easting = 3·1563371
Log difference Northing = 3·3208107

Log tan grid bearing = 9·8355264

∴ This figure is the log tangent of 34° 24′ 04″ *or* 145° 35′ 56″ *or* 214° 24′ 04″ *or* 325° 35′ 56″.

The grid bearing depends on which of the four quadrants AB lies in. In this case AB lies in the 2nd quadrant.†

∴ Grid bearing AB = 145° 35′ 56″.

---

* MAB in the second and third quadrants (Figs. 3 (*b*) and (*c*)).

† It is always desirable to draw a rough diagram to show in which quadrant the bearing lies.

Log difference Easting	3·1563371	Log difference Northing	3·3208107
Log sin grid bearing	9·7520354	Log cos grid bearing	9·9165079
	3·4043017	= Log AB =	3·4043028

Taking the mean of these two values of Log AB we obtain

$$\text{Log AB} = 3\cdot4043022$$
$$\therefore \text{AB} = 2536\cdot9 \text{ metres.}$$

2. From these formulæ the distance apart and mutual grid bearings of any two points can be calculated from their rectangular co-ordinates, or, if the co-ordinates of one point are known, the co-ordinates of a second can be computed from its distance and bearing from the first.

3. In carrying out these calculations, it is always advisable to draw a rough diagram sufficiently accurate to show in which quadrant the bearing lies.

In computing distances, it is advisable to work out the result by both of the formulæ (ii) and (iii). This involves very little extra trouble when logarithms are used, and gives a slight check against arithmetical and other errors, which might otherwise escape notice.

CHAPTER III.

TRIANGULATION.

8. General Description of Triangulation.

1. The process of triangulation consists in the observation of the angles of a series or network of triangles, in the calculation from these observed angles of the lengths of the sides of these triangles, and finally in the calculation from these lengths and the observed angles of the co-ordinates of individual points.

The angles are usually observed with an instrument known as a "Theodolite," described in Chap. IV.

The points on the earth's surface, at which the angles of a triangulation are measured, are called "trigonometrical stations." Since, however, when two angles of a plane triangle are known the third can be deduced, it is not always necessary to observe all three angles of a triangle to determine the lengths of its sides; there may be points in a system of triangulation at which the angles are not measured. Such points are called "intersected points."

2. The object of a triangulation is always to fix points for subsequent work, which may be either less accurate triangulation, traversing, or plane-tabling.

The first part of the process is, therefore, the selection of the points to be included in it. This will usually require a preliminary reconnaissance of the area, and must be done with due regard to economy and accuracy of the triangulation itself, and also to any subsequent work afterwards based on it.

Primary and secondary work forms the basis of triangulation of a less accurate order. The points fixed by such work will, for the most part, serve as stations of observation for subsequent work, which may be extended from it for considerable distances.

In minor and tertiary triangulation the points fixed are those required for detail survey. They do not necessarily serve as actual stations of observation for subsequent work, nor are extensions from them carried to considerable distances. In the case of the former the bulk of the points will be trigonometrical stations. In the case of the latter the majority may be intersected points.

3. Trigonometrical stations must be marked in such a manner that the theodolite can be centred over the exact point from which the angles are observed, and so that the mark is not likely to be lost through accidental disturbance or intentional destruction.

Observations must be taken to, as well as from, a trigonometrical station which will, therefore, usually consist of :—

(*a*) A buried mark stone, often supplemented by a protected surface mark.

(*b*) A signal or beacon.

Intersected points will usually be conspicuous features of the landscape, *e.g.*, church spires, windmills, factory chimneys, peaks of mountains, conspicuous trees, and the like.

4. Trigonometrical signals may be either luminous or opaque. The former are generally to be preferred for long rays and may be either sun-reflectors (heliotropes or heliographs), lamps, or some kind of rocket or flare.

Opaque signals are often called beacons and may be of many forms, such as tripods, flags, poles with a basket or brushwood tied to the top, &c.

Opaque signals, unless they are made very large, can seldom be seen at distances exceeding two or three miles unless they are silhouetted against the sky.

Luminous signals can be seen at great distances, but have the disadvantage that a man is necessary to operate them. Those which project a ray of light in a certain direction, such as a heliograph or signalling lamp, have also the characteristic that they can only be observed to from one point at a time. This is often a disadvantage for military purposes.

5. The number and arrangement of trigonometrical stations depend on the nature and purpose of the triangulation. In primary and secondary work the triangles are generally arranged in a series or chain, and the stations should be selected primarily with a view to giving figures of suitable size, and composed of " well-conditioned " triangles, *i.e.*, triangles which are as nearly as possible equilateral.

In minor and tertiary work, this is of less importance, though it should always be attempted. In this class of work the main consideration should be to keep the number of stations of observation as low as possible consistent with fixing the requisite number of points. Each intersected point should be fixed by at least three rays, and the deduced angle (*i.e.*, the apex angle) should not ordinarily be less than about 15°, though sufficiently accurate results can sometimes be obtained with smaller angles.

6. In minor and tertiary triangulation the " base " will generally be a side of a secondary series. When it is not possible to base such work on an existing triangulation, a base must be measured. When this has to be done, it is particularly important to select the sites for the base and the surrounding trig stations so that the triangles are well conditioned.

Fig. 4 shows a typical base extension. As a general rule, it is preferable to extend from a short base by well-conditioned triangles, than to try and measure too long a base. For minor triangulation the length of a base need not exceed 1 mile.

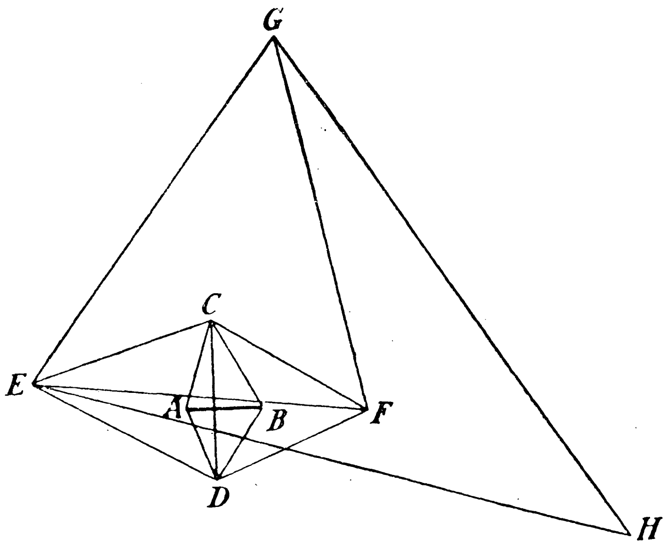


FIG. 4.

When a triangulation of any kind is executed as a series as opposed to a network, an attempt should always be made to arrange the triangles either as a double series, or a series of polygons (*see* Fig. 5).

Since such an arrangement lends itself better to the provision of the internal checks referred to in Section 2.

The size of the triangles will depend on the nature and object of the work. In primary triangulation they may be from 10 to 60 or more miles. In tertiary triangulation they will usually be from 1 to 5 miles.

7. In minor or tertiary triangulation it may sometimes happen that angles have to be observed from some point where it is impossible to set up the theodolite. In such cases the angles may be observed from some other point close by and the angles afterwards reduced to the true values at the proper point. Such a station is called a " satellite station " (*see* Chap. IV, Sect. 17).

9. Preliminary Reconnaissance for Triangulation.

1. The object of a preliminary reconnaisance is—

(*a*) To decide on the stations of observation and the figures formed by them.

(b) To mark the exact site of each station, and to erect any beacons and signals necessary.
(c) To select the intersected points.

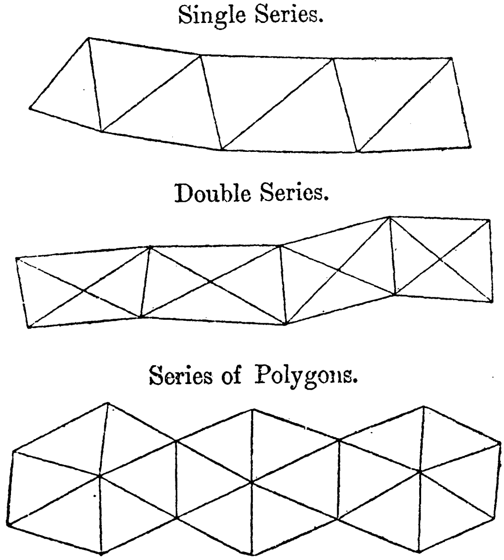


FIG. 5.

The reconnaissance is most conveniently carried out by plane table on a scale about one half or one quarter of that of the detail survey for which the triangulation is intended.

2. When maps of the area exist, much time can be saved by studying the map and selecting various possible arrangements of figures before going on to the ground. An itinerary should be prepared, and all likely positions for trig stations visited in turn. The plane table should be set up at each of these, and also at intermediate points so as to fix as many intersected points as possible, even though some of these may afterwards be found to be invisible from the stations.

3. When the reconnaissance is complete, the chart or diagram should be examined to see whether any of the projected stations can be dispensed with, bearing in mind that three rays are sufficient to fix an intersected point.

10. Marking of Trigonometrical Stations.

1. Various types of beacons are shown in Plates I to IV. When luminous signals are employed, a programme of the observations should be prepared before commencing observing. In this programme should be included the movements of the men working the signals.

A simple code of signals should be arranged with them, so that each can be notified by signal from the observing station in occupation that work at that station is complete.

The signaller, from his programme, should then know to which point he should next direct his light, or whether he has to proceed to

PLATE I.

QUADRIPOD BEACON.

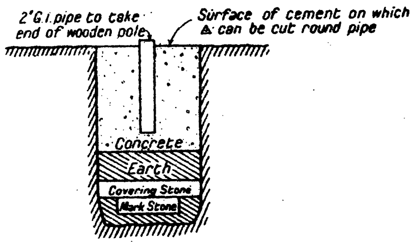
Used on the major triangulation of the East Africa Protectorate (*see also* Plate III). The wind and sun screen has been dropped on one side in order to have a clear view of the theodolite.

IRON QUADRIPOD BEACON, WITH WOODEN BATTENS.

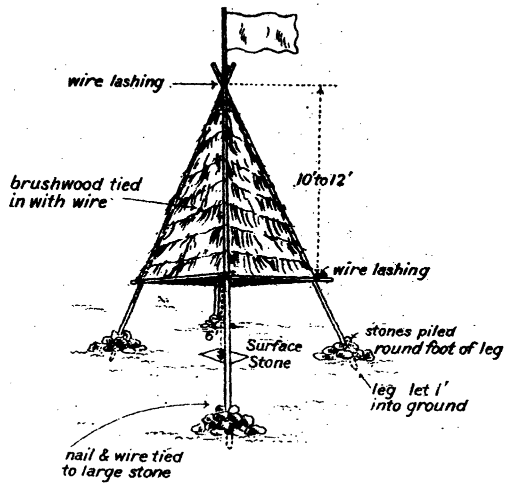
Used on the major triangulation of the East Africa Protectorate for base extension points only. The beacon was not erected until the observer, having completed his short rays, and his long rays from the point, was leaving the point to observe long rays *to* it. The short rays were observed to lamps at night. The calico is shown tied up during observations; when unfolded it extended from the battens to the ground.

To face page 22.

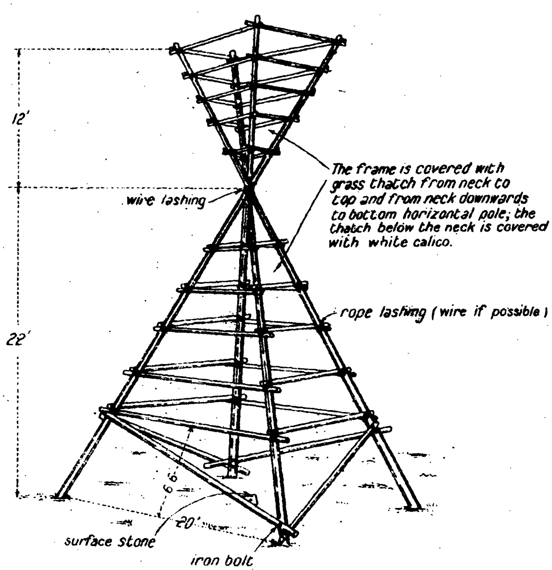
PLATE II.



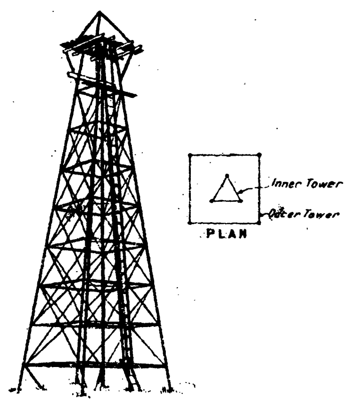
MARK USED ON
ORANGE FREE STATE TOPO. SURVEY.



BEACON USED ON ORANGE FREE STATE SURVEY.



FRAME OF BEACON USED ON
MAJOR TRIANGULATION EAST AFRICA (see Plate IV).



TOWER STATION USED ON
TOPO. SURVEY CANADA.

Malby & Sons. Lith.

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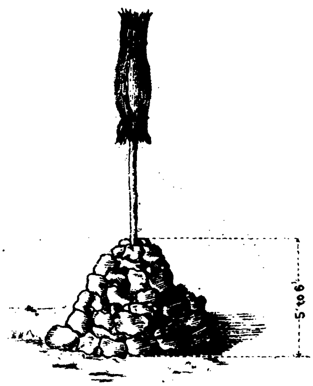
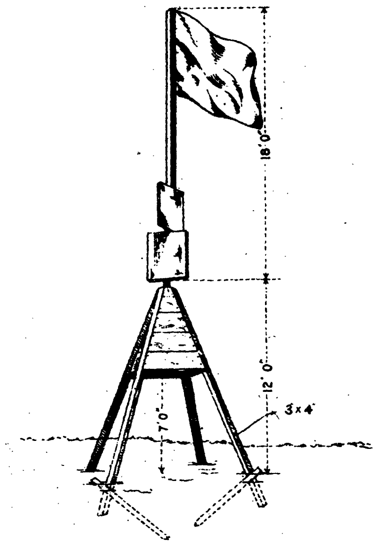
PLATE III.

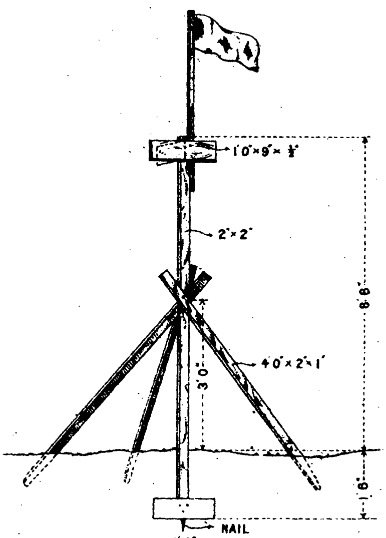
TRIGONOMETRICAL SIGNALS FOR SHORT DISTANCES



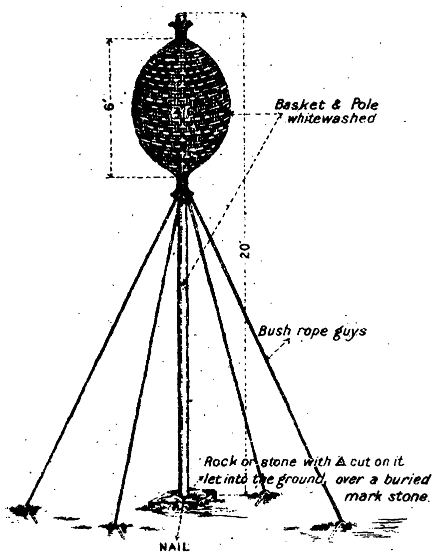
INDIAN POLE AND BRUSH SIGNAL



QUADRIPOD SIGNAL



ORDNANCE SURVEY TRIGONOMETRICAL POLE



WEST AFRICAN BASKET SIGNAL

Malby & Sons, Lith.

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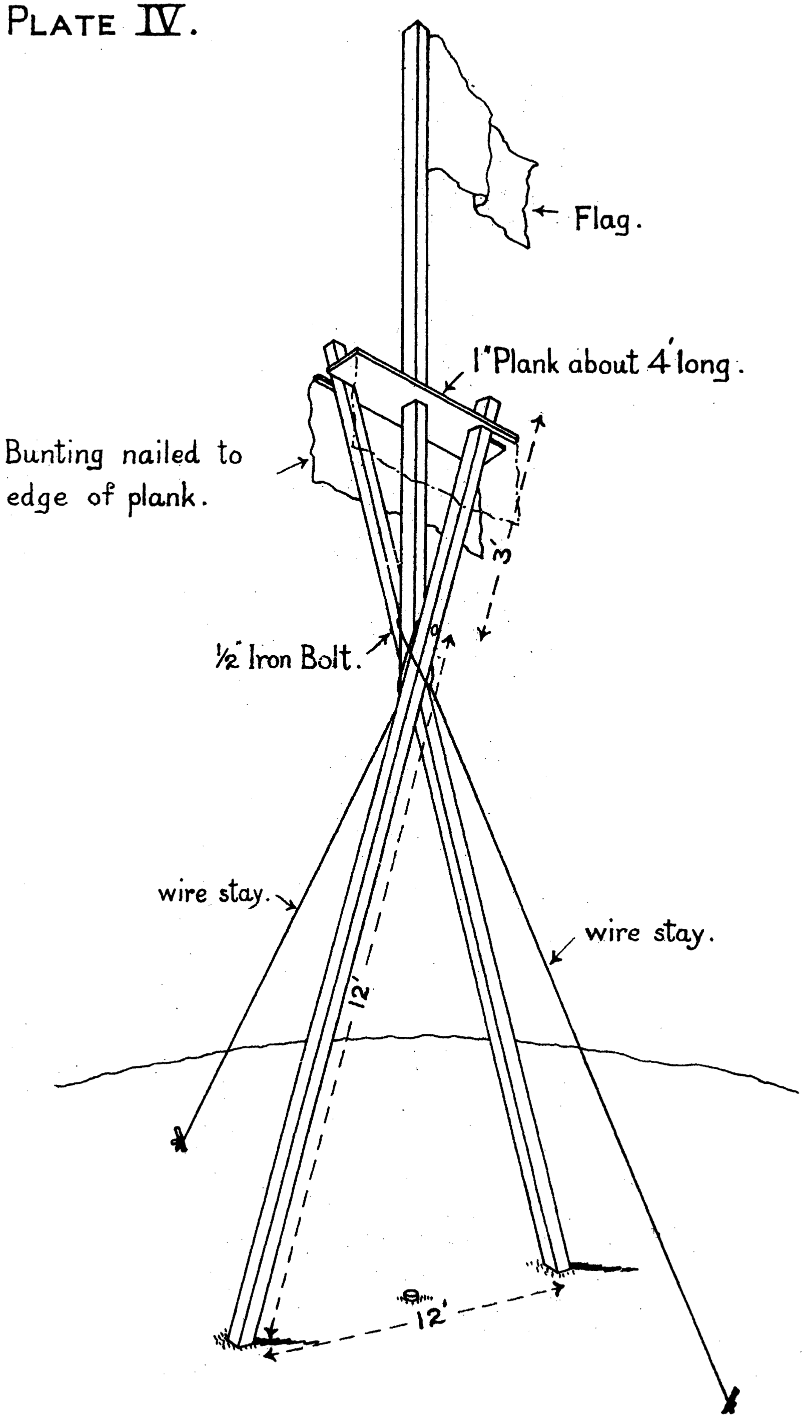
PLATE IV.



Collapsible Bipod Beacon constructed of 2"x 2" Timber.

Malby & Sons, Lith.

another station. As the number of signallers available is always limited, the preparation of this programme requires some care to ensure that no time is lost in getting the signallers into their proper positions for observation at each successive station.

2. For artillery work it will seldom be necessary to observe very long rays. Such triangulation as may be required for artillery purposes will most probably be for survey of sound ranging bases, or for fixing points in our own forward areas, or in the enemy's lines. Such work will often have to be done against time, and it may be necessary to employ beacons or signals, which allow observations at adjoining stations to be made at the same time. Signals such as a flag or pole and brush are inconvenient in such cases, as, if they are used, some of the stations must be observed as satellites (*see* Sect. **17**).

Tripod or quadripod signals are convenient, but are generally rather bulky and heavy to carry about, and not very easy to erect.

A " bipod " beacon such as that shown in Plate IV is easy to construct, and erect, and is reasonably portable.

If surmounted by a flag of white calico 3 feet square, it can be seen in clear weather up to a distance of 4 or 5 miles.

3. In the case of an advance, following an action, into unsurveyed country it may be necessary to beacon the trig stations behind the line of departure, so that they can be seen for long distances and from many directions.

Steel observation towers, such as the " Inglis Tower " employed in France in 1918, can be erected to a height of 150 or 200 feet in one day and make very conspicuous marks.

The " vertical light ray " is another device which might be useful for this purpose, but is still in the experimental stage. Its object is to provide a suitable mark to which observations can be made from any direction. The apparatus may take the form of a rocket or light, which can be fired vertically into the air, leaving a trail of smoke or flame, or, more probably, of a narrow beam of light pointed vertically upwards and made visible, if necessary, by firing smoke bombs into the air, so that the smoke drifts across the beam.

It is possible, also, that searchlights pointed vertically upwards might be useful for night work, as they can be seen for great distances; while by day kite balloons, whose positions have been fixed, might be made use of.

CHAPTER IV.

THE THEODOLITE.

11. General Description.

1. The theodolite is an instrument designed for the measurement of horizontal and vertical angles, and consists essentially of a telescope, carried on trunnions, fitting into a carrier which can be revolved about a vertical axis.

The telescope can thus be revolved in either a vertical or a horizontal plane. Moving with it are two pointers travelling over graduated arcs engraved on circular plates, one vertical and the other horizontal, fixed to the base or body of the instrument.

The graduated arcs are known as "limbs." Theodolites are designated by the diameter of the horizontal limb, which may be from 3 inches to 5 inches for tertiary work, or up to 12 inches for primary triangulation.

2. To measure an angle between two points, the telescope is laid first on one and then on the other by means of a crosshair appearing in the field of view. The reading of the pointer on the limb is noted in each case, and the difference between the two readings gives the angle between the points.

3. In measuring an angle thus, it is not possible to make use of graduations of more than a certain degree of refinement, and the pointer will therefore seldom point to an exact division on the scale.

Verniers and micrometers are two devices which are most commonly used for interpolating between the scale divisions to obtain an exact reading.

12. Verniers and Micrometers.

1. *Principle of the vernier.*—If a length of, say, three divisions of the scale be divided into four, as shown in Fig. 6, it is evident that the length of each of the latter is three-quarters of a primary division, and that the difference in length between one division on each scale is one-quarter of the length of a division on the primary scale.

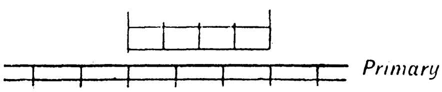


Fig. 6.

The same result is obtained if five primary divisions are divided into four in the vernier scale.

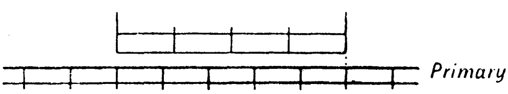


Fig. 7.

2. Generally, if $n - 1$ or $n + 1$ primary divisions are divided into n divisions on the vernier, the difference between the length of the primary and a vernier division will be $1/n$ of a primary division.

3. It can be seen from Fig. 7 that, if the vernier scale be moved to one side by a distance equal to a quarter of a primary division, the effect will be to make the third division on the vernier correspond exactly with one of the divisions (the fourth from the starting point) on the primary.

In Fig. 6 a similar movement will make the first vernier division correspond with a primary division in the same way.

By simply noting which division of the vernier exactly corresponds with one of the primary divisions, it is thus possible to say how far, in multiples of the difference between the length of a division on each scale, the end of the vernier scale has been moved to one side or the other.

4. The application of this principle can be seen from Fig. 8. In this the primary scale is graduated in degrees and sixths of a degree, each division represents 10 minutes.

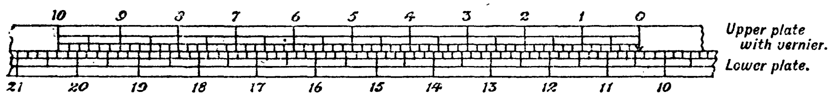


FIG. 8.

Sixty-one of these divisions are divided into 60 on the vernier. The difference in length of one division on each scale represents therefore 10 minutes.

The figures of the vernier scale represent minutes. The reading of the pointer, forming the end graduation of the vernier scale is thus 8 minutes 20 seconds from the second division to the left of 10 degrees.

The reading of the scale is therefore 10° 28′ 20″.

5. Sometimes two adjoining divisions will appear to coincide. In such cases the reading should be taken as the mean of the two.

In the above example it might be said that 8′ 10″ is as near coincidence as 8′ 20″, and the reading might be taken as 8′ 15″.

In practically all theodolites a small microscope or magnifying glass is provided for reading the limb.

6. *The micrometer microscope.*—In place of the vernier the microscope may be provided with a pair of cross wires close together, which can be moved from side to side across the field of view by means of a slow motion screw.

When the microscope is focussed, the wires and the limb graduations appear as in Fig. 9. The pointer of the vernier is usually replaced by a small notch.

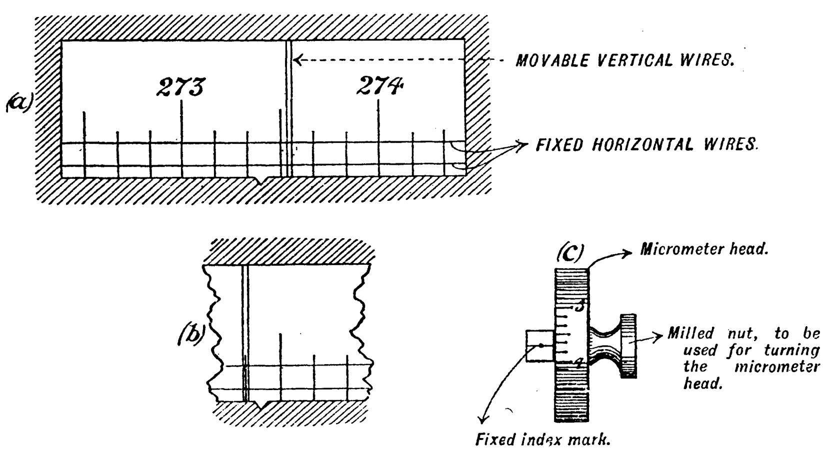


FIG. 9.

The slow motion screw is provided with a "micrometer" head, and its pitch is chosen so that one complete revolution of the micrometer moves the cross wire laterally through a distance exactly equal to one division on the primary scale.

The micrometer head is graduated and a fixed index mark or pointer fixed beside it.

To obtain the reading of the notch the milled nut attached to the micrometer head is turned until the cross wires enclose an adjoining graduation on the limb. Minutes and seconds, &c., can then be read off on the micrometer head.

In the example given the reading is 273° 24′ 17″.

7. *Micrometers* are generally easier and quicker to read, and are also more accurate than verniers, but are liable to be thrown out of adjustment if the theodolite is roughly handled. Provided, however, that reasonable care is taken in handling and transporting the instruments, they should give no trouble.

13. Examination of a Theodolite.

1. Theodolites are always issued packed in wooden cases and provided with a tripod stand.

A theodolite, on being first taken over, should be examined to see that it is in good order.

The stand should be first set up and inspected to see that there is no play in the head or in the metal shoes at the foot of the legs.

The box should then be opened, and the positions of all parts of the instrument carefully noted.

The positions of the object and eye ends of the telescope should be marked on the box, and the way in which the tangent screws fit in especially observed.

14. Setting up of a Theodolite.

1. Before measuring angles with a theodolite the instrument must first be accurately centred over the station mark, and then levelled.

The stand must be placed with the legs well apart, and pressed sufficiently into the ground to give a steady base.

It is absolutely essential that no movement of the stand should take place during the observations.

2. When the stand has been placed in position as nearly as possible level and over the mark, the theodolite should be removed from its box, and placed on the stand, care being taken to remove dust, &c., from the tribrach before the foot screws are placed in position.

The plumb-bob should then be hooked to the base of the instrument, and the legs of the tripod moved about until the bob hangs exactly over the station mark, and the top of the stand is sufficiently level for the final levelling of the instrument to be effected by means of the foot screws.

3. *Levelling of the theodolite.*—Place the longer of the two levels on the base plate parallel to any two of the foot screws. Turn these two screws until the bubble is in the centre of its run. Then turn the other foot screw until the smaller or cross bubble is also in the centre of its run.

Now turn the telescope through 180° in azimuth, and allow the bubble of the longer level to come to rest. If it has moved its position, note the position it now takes up, and by means of the first two foot screws bring it back *half-way* to its centre position.

Again turn the telescope through 180° in azimuth, and note the position taken up by the bubble. If the bubble has been correctly brought to the " half-way " position after the first movement, it should be found unchanged on bringing the telescope back to its first position. If it has moved, it should again be brought by means of the same screws to a position half-way between its second and third positions, and so on until a position is found such that the bubble remains unmoved when the telescope is moved through 180°.

Now turn the telescope through 90°, and repeat the process by turning the third foot screw only.

This process must be continued, using first one pair of foot screws together and then the third, until the position of the *bubble remains unchanged* whatever the position of the upper plate.

15. Adjustments of a Theodolite.

(1) *Occasional adjustments.*

(*a*) Micrometer adjustments.
(*b*) Collimation in altitude and azimuth.
(*c*) Verticality and horizontality of the cross wires.
(*d*) Adjustment of levels.
(*e*) Adjustment of transit axis.

(2) *Adjustments before commencing each observation* (*station adjustments*).

(*a*) Centreing.
(*b*) Levelling.
(*c*) Distinct vision of wires.
(*d*) Parallax in focus.

3. Adjustments of the first class should generally be made by an artificer, but every trigonometrical surveyor should know how to test his theodolite to see that it is correctly adjusted.

4. *Testing of micrometers.*—Micrometers should be tested by traversing the wires from one division of the scale to the next and seeing that the reading on the micrometer head remains unchanged.

5. *Collimation in azimuth.*—Set up and level the theodolite, and intersect a well-defined object on about the same level as the observer with the vertical wire, and read the horizontal limb.

Now revolve the telescope through 180° about the transit axis (this is known as " changing face "), release the upper plate, and again intersect the object with the vertical cross wire. The reading on the horizontal limb should differ from the first by exactly 180°.

Collimation in azimuth may also be tested by changing pivots, *i.e.*, by intersecting an object as above, removing the telescope gently from its pivots and replacing it upside down—that is, with the object end still pointing towards the object but with the pivots interchanged.

6. *Collimation in altitude.*—Level the instrument carefully, and

bring the bubble attached to the vertical limb to the centre of its run by means of the clip screws.

Intersect an object as before but with the horizontal wire, and read the vertical limb. Change face, and again intersect the object. The second reading should be the same as the first.

An error of collimation in altitude or azimuth is in practice eliminated by making all observations on both faces of the instrument, and taking the mean of each pair of readings.

7. *Verticality and Horizontality of the cross wires.*—Verticality or horizontality of the wires is easily tested by carefully levelling the instrument and intersecting a well-defined object with the cross wire.

As the telescope is moved in azimuth the object should appear to move along the horizontal wire. As the telescope is moved in altitude it should move in a similar manner along the vertical wire.

This method must be carried out with great care to avoid jarring or shaking the instrument, and thus vitiating the observation.

8. *Adjustment of the transit axis.*—The level of the transit axis is most easily tested with a striding level, but it may also be done as follows :—

Set up the theodolite facing a high wall, such as the side of a house, and very carefully level the base plate.

Select some well-defined point on the wall at an elevation of between 30 and 60°, and intersect it with the vertical wire of the telescope.

Clamp the base plates, bring the telescope horizontal, and make a mark on the foot of the wall at a point intersected by the vertical cross wire just above or below the horizontal wire.

Change face and again intersect the upper mark.

Clamp both base plates and again bring the telescope horizontal.

If the transit axis is level the mark should be bisected by the vertical wire.

9. *Focus and parallax.*—Before commencing any observation the telescope should be adjusted for focus and parallax.

The telescope should be pointed to the sky, and the eyepiece screwed in or out until the cross wires are seen with perfect distinctness.

The telescope should now be pointed at some well-defined distant object, and the object glass screwed in or out until distinct vision is obtained.

Now move the eye slowly from side to side, and note if the object appears to move relatively to the cross wires. If no movement can be detected, the focus is correct. If any movement is apparent, it is due to imperfect focus or *parallax,* which must be eliminated before observation is commenced.

Note whether the object moves with the eye or against it. In the first case the object glass must be screwed in, and in the second case moved out, until the correct focus is obtained.

It should be noted that the correct setting of the *eyepiece* depends on the eyesight of the observer, while the focus or setting of the object glass depends on the distance of the object. It is practically constant, however, for all objects at a greater distance than about 800 yards from the observer.

16. Use of the Theodolite.

1. All observations of angles with a theodolite should be made on a regular system or routine, and the results "booked," whenever possible, in special "angle books" printed or ruled up for the purpose beforehand.

2. Observations to trigonometrical stations, and to intersected points, should, as a rule, be made separately (*see* para. 9).

3. As soon as the instrument has been set up, centred and levelled, the height of the transit axis above ground level should be measured and booked.

4. The observer should then examine the diagram of the triangulation, select one of the points to be observed as a zero or reference point, identify all the other points, and then write down their names or numbers in sequence, either clockwise or counter-clockwise, in the angle book.

5. In Fig. 10, assuming the observer is at B, and that it is required to observe horizontal angles to C, D, and E, the procedure is as follows :

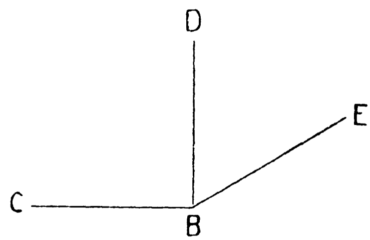


Fig. 10.

With the telescope at "face left," *i.e.*, with the vertical circle on the left of the observer as he looks into the telescope, set the A vernier or micrometer to about 0° 4′, and clamp the upper plate to the lower. Release the lower plate, and turn the whole instrument until the reference object, say, C, comes into the field of view. Clamp the lower plate, and intersect C with the tangent or slow motion screw. Release the upper plate, and turn the telescope completely round counter-clockwise until C again comes into the field of view, taking care that the vertical wire does not overshoot the point. Clamp the upper plate, and complete the intersection of C with the tangent screw of the upper plate.

Read both microscopes, and enter the readings in the angle book.

Release the upper clamp and turn the telescope counter-clockwise towards E, which should be intersected in the same way. Read both microscopes and book the readings.

The same process must be repeated for D and any other points to be observed. Finally, the first station C should always be intersected again, and the readings recorded, to see that no movement of the lower plate has occurred during the round.

6. With a 5″ or 6″ vernier instrument the last reading should not differ from the first by more than 30″. If the difference exceeds this amount, the observation should be rejected and repeated.

A round executed as above is described as "face left, swing left, zero 0°."

The face should now be changed, and the observations repeated in a similar manner but in a clockwise rotation, *i.e.*, "face right, swing right."

For tertiary work, 2 zeros, with two faces on each zero, one swing on each face, should be sufficient.

(In primary work the number of zeros may be 6 or more with two faces on each zero and two swings on each face.)

7. The second and subsequent zeros should be chosen so as to obtain all the readings as far as possible on different parts of the arc.

Thus, for the second zero, the plates may be clamped together with the A vernier reading 45°.

The vernier should not be clamped exactly at 0° or 45°, as it may happen that, when the reference object is intersected for the second or third time, the reading may be a few seconds less than the zero, *e.g.*, 359° 59′ 50″, a figure which would be troublesome when taking out the final angles.

8. *Recording and abstracting of angles.*—A specimen page of an angle book, to show the way in which angles should be booked, is given below.

The abstraction of angles consists in :—

(*a*) Taking the mean of the vernier readings for each microscope on each face.

(*b*) Subtracting the zero reading from the other readings.

(*c*) Collecting and taking the mean of the resulting observations to each object.

Specimen page of an "Angle Book."

Station ...	... *E.*	Instrument ...	... *Watts 5″T.V. No. 584.*
Description	... *Tripod.*		
R.O. ...	... *A.*	Height of Instrument	*1·51 metres.*
Observer ...	... *Lieut. Green.*	Compass Bearing of R.O.	179° 30′.
Booker ...	... *Corpl. T. White.*	Date ...	... *15.8.1922.*

Station.	Face and Swing	A Vernier.			B Vernier.		Mean of A and B.			Reduced Angle.		
		°	′	″	′	″	°	′	″	°	′	″
A	*R*	*00*	*03*	*00*	*02*	*30*	*00*	*02*	*45*	*00*	*00*	*00*
B		*27*	*31*	*00*	*30*	*00*	*27*	*30*	*30*	*27*	*27*	*38*
C		*48*	*43*	*30*	*43*	*00*	*48*	*43*	*15*	*48*	*40*	*23*
D		*104*	*22*	*00*	*21*	*00*	*104*	*21*	*30*	*104*	*18*	*38*
A		*00*	*03*	*30*	*02*	*30*	*00*	*03*	*00*	*00*	*00*	*00*
A	*L*	*180*	*04*	*00*	*04*	*30*	*180*	*04*	*15*	*00*	*00*	*00*
D		*284*	*21*	*30*	*22*	*00*	*284*	*21*	*45*	*104*	*17*	*45*
C		*228*	*43*	*30*	*43*	*30*	*228*	*43*	*30*	*48*	*39*	*30*
B		*207*	*32*	*00*	*32*	*30*	*207*	*32*	*15*	*27*	*28*	*15*
A		*180*	*03*	*30*	*04*	*00*	*180*	*03*	*45*	*00*	*00*	*00*
A	*L*	*225*	*08*	*00*	*07*	*00*	*225*	*07*	*30*	*00*	*00*	*00*
D		*329*	*26*	*30*	*26*	*00*	*329*	*26*	*15*	*104*	*19*	*00*
C		*273*	*47*	*30*	*46*	*30*	*273*	*47*	*00*	*48*	*39*	*45*
B		*252*	*36*	*00*	*35*	*00*	*252*	*35*	*30*	*27*	*28*	*15*
A		*225*	*07*	*30*	*06*	*30*	*225*	*07*	*00*	*00*	*00*	*00*
A	*R*	*45*	*06*	*00*	*06*	*30*	*45*	*06*	*15*	*00*	*00*	*00*
B		*72*	*33*	*30*	*34*	*00*	*72*	*33*	*45*	*27*	*27*	*30*
C		*93*	*46*	*30*	*46*	*30*	*93*	*46*	*30*	*48*	*40*	*15*
D		*149*	*23*	*30*	*24*	*30*	*149*	*24*	*00*	*104*	*17*	*45*
A		*45*	*06*	*00*	*06*	*30*	*45*	*06*	*15*	*00*	*00*	*00*

ABSTRACT OF ANGLES.

B			C			D			Final Angles.			
									Station.	Angle.		
°	′	″	′	°	″	°	′	″		°	′	″
27	*27*	*38*	*48*	*40*	*23*	*104*	*18*	*38*	*A*	*00*	*00*	*00*
	28	*15*		*39*	*30*		*17*	*45*	*B*	*27*	*27*	*54*
	28	*15*		*39*	*45*		*19*	*00*	*C*	*48*	*39*	*58*
	27	*30*		*40*	*15*		*17*	*45*	*D*	*104*	*18*	*17*
	4)111	*38*		*4)159*	*53*		*4)73*	*08*				
	27	*54*		*39*	*58*		*18*	*17*				

9. The observation of intersected points should be effected in a similar manner to that of the trig stations, except that only one zero, with one swing on each face, is required. A trigonometrical station must be included in the round, preferably as the reference point.

10. In making observations to objects which subtend a considerable angle at the theodolite, such as a tree, it is generally sufficient to intersect the centre of the object as well as it can be estimated.

In observing houses it is better to take either the gable end, or a chimney on one particular corner, which should be specified.

When a preliminary reconnaissance of the area has been made, and a diagram prepared, points should be numbered and referred to in the angle book by their numbers. Descriptions of all points should be kept in a separate note book, or entered on the chart.

11. *Observation of vertical angles.*—Vertical angles should be observed with the horizontal plate unclamped. It is unnecessary to observe on any particular swing, but it is generally convenient to intersect the object with a portion of the horizontal cross wire just to one side of the vertical wire, and to use the same portion of the wire after changing face, remembering that, if it was on the left of the centre before changing, it will be on the right after changing face.

12. The readings of the vertical circle bubble as well as the limb itself should be recorded, or, alternatively, the bubble should be brought to the centre of its run by the clip screw before each observation.

The vertical circle bubble is generally very sensitive, and, unless the theodolite has been levelled with very great care, it may be found that small movements of the bubble take place as the telescope is turned in different directions.

These must be entered under the headings O (Object end) and E (Eye end), and the recorded readings of the limb corrected afterwards thus :—

Suppose that two observations are taken to a point, one F.L., and one F.R., and the readings are as follows :—

	O	E
F.L.	5	8
F.R.	7	6
Sum	12	14

In this case the sum of the readings of the eye end (E) exceeds that of the object end (O) by 2. The number of the readings of the bubble is 2. To find the dislevelment in terms of divisions of the scale, 2 must be divided by twice the number of readings, *i.e.*, by 4. Suppose the value of one division of the scale is 16 seconds—then the correction is $16 \times (2 \div 4)$. That is 8 seconds. This correction must be applied with the proper sign. In this case the eye end being in excess, the zero from which measurements were made was not the horizontal but a line whose depression was 8″. All elevations will therefore be read too large by this amount and all depressions too small. If the object end had been in excess, the correction would be added to elevations and subtracted from depressions.

$$\text{Generally, correction to elevation} = + \frac{O - E}{2 \times \text{No. of readings}} \times \text{value of 1 division}$$

$$\text{and to depression} = - \frac{O - E}{2 \times \text{No. of readings}} \times \text{value of 1 division.}$$

13. The value of one division of the scale can be determined thus :— By means of the clip screws bring the bubble up to one end of its run (*e.g.*, object end) so that the *object* end of the bubble corresponds approximately with the extreme reading of the scale. Intersect some convenient object with the horizontal wire. Read and record one end of the bubble (*e.g.*, object end) and the vertical angle. By means of the clip screws bring the bubble back towards the eye end as far as possible, care being taken that it is really floating, and within the graduations of the scale. Intersect same object as before, and record the vertical angle and the reading of the object end of the bubble. The difference between the two readings of the object end of the bubble gives the dislevelment in terms of divisions of the scale, and the difference between the two vertical angles gives the same dislevelment in minutes and seconds of arc. Dividing this angular measurement

by the number of divisions of dislevelment, the value in arc of one division of the scale is obtained. Thus :—

	Elevation.			Object end of bubble.
	°	′	″	
1st Observation	7	3	28	18
2nd Observation ...	7	0	00	4
Difference		3	28	14

$$\text{Value of one division} = \frac{208''}{14} = 14{\cdot}86''.$$

14. Vertical angles should, when possible, be observed during the hours of minimum refraction, *i.e.*, between 13.45 and 15.45 hours; this practice ensures also that all vertical angles are observed about the same time. This is of importance, in that it is generally assumed, when reciprocal vertical angles have been taken from two stations, that the refraction is the same at each.

Vertical angles taken during the early hours of the morning or late in the evening, when the refraction is great, are almost useless, unless vertical angles are observed at the same time to trig stations at varying distances whose heights are known, so that the refraction can be computed.

From the above it will be seen that vertical and horizontal angles should not, if it can be avoided, be observed simultaneously by intersecting the object with the cross wire. Sometimes, when time is limited, this cannot be avoided, but the results will be less accurate than those obtained from separate observations.

15. *Observations at a satellite station.*—When observing at a satellite station, the theodolite should be set up as close as convenient to the station mark, and the mark should be included in one of the rounds of angles taken. The distance of the theodolite from the mark should be measured and entered in the angle book.

16. *General remarks on observations.*—At every trigonometrical station the magnetic bearing to one trig station should be observed and recorded.

At the end of each set of observations the observer should examine the recorded angles for obvious errors. Before leaving the station the abstract of angles should be made out to see that no re-observation is necessary.

17. It saves time to have a second man for booking the angles read by the observer. The abstract can be made out by this man as the observations proceed, and any discrepancies at once brought to notice.

18. A description of each station, its exact position with reference to some local landmark, and a description or drawing of the beacon should be entered in the angle book before leaving the station.

EXAMPLE OF BOOKING OF VERTICAL ANGLES.

Place	*C.*	Instrument	*No. 641.*	Page 7.
Date	*23.7.22.*	Level Value	*1 Div.= 20″.*	
Weather ...	*Fine.*	Height of Signal ...	*7 metres.*	
	Visibility good.	Height of Instrument ...	*1 metre.*	

Object	Face and Swing	Vertical Readings.			Uncorrected Vertical Angles.	Level.		Corrected Vertical Angles.
		—	—	Means.		E	O	
		° ′ ″	′ ″	° ′ ″	° ′ ″			° ′ ″
A	*L*	*4 30 00*	*30 10*	*4 30 05*	*4 30 10*	*5*	*7*	*4 30 20*
...	*R*	*175 29 40*	*29 50*	*175 29 45*		*8*	*4*	
						13	*11*	
						Corrn.= + 10″.		

Observer ... ... *X.* Recorder ... ... *Y.*

17. SATELLITE STATIONS.

1. The angles at a satellite station are observed in the ordinary way, and the signal over the mark, or the mark itself, must be included in the round.

The distance of the station mark or signal from the satellite station must be measured, and recorded in the angle book. The satellite station should be selected so that this distance is as short as possible.

2. In general two cases of satellite stations occur—

(i) When it is found on visiting a station that the signal has been incorrectly centred, or the wrong signal has been observed from one or more other stations.

(ii) When it is found to be impossible or inconvenient to set up a theodolite exactly over the station mark.

In the first case, the rays observed to the wrong or incorrectly centred beacon have to be connected to bring them on to the correct position, and in the second case they have to be reduced to the values which would have been observed from the station mark, had it been possible to set up a theodolite there.

3. This is called " reduction to centre " and is done as follows :—

In the first case—

In Fig. 11 let A and B be two stations from which an incorrect beacon T has been observed, S being the correct position. It is required to find the angles SAB and SBA. The angles AST, BST, TAB, and ABT being known and the distance ST measured.

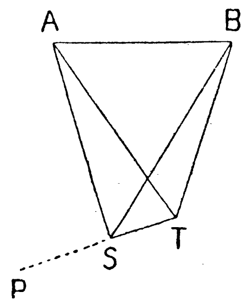


Fig. 11.

$$\text{Then SAB} = \text{TAB} + \text{SAT}$$

$$\text{and sin SAT} = \frac{\text{ST}}{\text{AT}} \text{ sin AST.}$$

AT can be computed from the triangle TAB, of which the side AB and the angles TAB and ABT are knnow.

Similarly,

$$\text{SBA} = \text{ABT} - \text{SBT}$$

$$\text{and sin SBT} = \frac{\text{ST}}{\text{BT}} \text{ sin BST.}$$

In the second case—

In Fig. 11 the angle ATB is observed and the angle ASB is required.

$$\text{BSP} = \text{BTS} + \text{SBT}$$
$$\text{ASP} = \text{ATS} + \text{SAT}$$

∴ subtracting

$$\text{ASB} = \text{ATB} + \text{SBT} - \text{SAT}$$

$$\text{sin SBT} = \frac{\text{ST}}{\text{SB}} \text{ sin BTS}$$

$$\text{sin SAT} = \frac{\text{ST}}{\text{SA}} \text{ sin ATS.}$$

The lengths of SA and SB need only be computed approximately or may be measured from the map or chart.

4. SBT and SAT should always be small angles and can be conveniently computed thus :—

Log ST (feet) = ...
Log cosec 1″ = ...
Colog SB = ...
Log sin BTS = ...

Log seconds SBT =

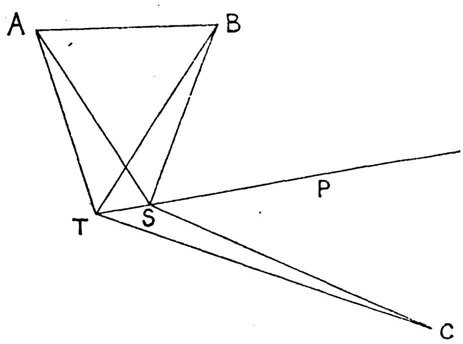
5. When a number of angles have to be reduced to centre it saves writing if log ST and log cosec 1″, which are the same for each angle, are added separately, thus :—

If, log ST + log cosec 1″ = K, say,

then each angle is computed thus :—

	SAT	SBT	SCT	SDT
K				
Colog S	SA	SB	SC	SD
Log sin S	ATS	BTS	CTS	DTS
Log seconds ...	SAT	SBT	SCT	SDT

6. In practice, when a round of angles comprising a large number of points has to be reduced to centre, the simplest plan is first to refer all the angles to the point S as zero or reference point (*see* fig. 12).



The angles have been observed at T.

It is required to find the angles at S.

FIG. 12.

Then compute the angles SAT, SBT, etc., from the formulæ given above.

The reduction to S as zero gives the angles PTA, PTB, etc., and the required angles PSA, PSB, etc., are obtained by adding the computed angles, *e.g.* :

PSA = PTA + SAT
PSB = PTB + SBT, &c.

(Note, however, that when the angle exceeds 180° the computed angle must be subtracted, *e.g.* :

PSC (measured clockwise) = PTC — SCT.

After adding or subtracting these angles, the round may be left with the line TSP as zero ; though it can, if desired, be reduced to any other zero, say, A, by subtracting PSA from all.

It does not matter that P is an imaginary point, since the angles required for computation are obtained by subtracting one reading from another.

EXAMPLE. COMPUTATION OF SATELLITE. REDUCTION TO CENTRE.

Round as observed.				Distance in Metres.	Reduced to Station Mark as zero.			Correction.		Corrected reading Reduced to Centre.		
								′	″			
C	0	0	0	1930	81	50	00	+ 5	17	81	55	17
B	275	16	38	2270	357	06	38	− 0	14	357	06	24
D	316	08	58	4150	37	58	58	+ 1	32	38	00	30
Station Mark	278	10		3	0	0	0					
Points.												
1.	114	26	15	1200	196	16	15	− 2	24	196	13	51
3.	257	47	18	1600	339	37	18	− 2	15	339	35	03
2.	261	58	11	640	343	48	11	− 4	29	343	43	42
21.	272	21	21	2370	354	11	21	− 0	26	354	20	55
5.	301	05	06	1620	22	55	06	+ 2	29	22	57	35
7.	319	48	48	2300	41	38	48	+ 2	59	41	41	47
9.	338	27	11	3050	60	17	11	+ 2	56	60	20	07
6.	336	14	31	1640	58	04	31	+ 5	20	58	09	51
4.	353	33	11	640	75	23	11	+15	35	75	38	46

log 3 = *0·4771*
log cosec 1″ = *5·3144*

Sum = *5·7915* = *K*.

C.	B.	D.
K = *5·7915*	K = *5·7915*	K = *5·7951*
colog 1930 = $\bar{4}$*·7144*	colog 2270 = $\bar{4}$*·6438*	colog 4150 = $\bar{4}$*·3819*
L sin 81°·50 = $\bar{1}$*·9956*	L sin 357.06.38 = $\bar{2}$*·7025*	L sin 37.58.58 = $\bar{1}$*·7892*
sum = *2·5015*	sum = *1·1378*	sum = *1·9626*
= *317·3″*	= *13·7″*	= *92″*
+ *5′ 17″*	− 14″	+ 1′ *32″*

Similarly for points.

1.	3.	2.	21.	5.	7.	9.	6.	4.
5·7915	*5·7915*	*5·7915*	*5·7915*	*5·7915*	*5·7915*	*5·7915*	*5·7915*	*5·7915*
$\bar{4}$*·9208*	$\bar{4}$*·7958*	$\bar{3}$*·1937*	$\bar{4}$*·6252*	$\bar{4}$*·7904*	$\bar{4}$*·6382*	$\bar{4}$*·5156*	$\bar{4}$*·7851*	$\bar{3}$*·1937*
$\bar{1}$*·4474*	$\bar{1}$*·5419*	$\bar{1}$*·4455*	$\bar{1}$*·0054*	$\bar{1}$*·5904*	$\bar{1}$*·8225*	$\bar{1}$*·9388*	$\bar{1}$*·9288*	$\bar{1}$*·9857*
2·1597	*2·1292*	*2·4307*	*1·4221*	*2·1723*	*2·2522*	*2·2459*	*2·5054*	*2·9709*
144″	*135″*	*269″*	*26″*	*149″*	*179″*	*176″*	*320″*	*935″*
− 2′ 24″	− *2′ 15″*	− *4′ 29″*	− *26″*	+ *2′ 29″*	+ *2′ 59″*	+ *2′ 56″*	+ *5′ 20″*	+*15′ 35″*

CHAPTER V.

BASE MEASUREMENT

18. Methods of Base Measurement.

1. If the angles of a triangulation are correctly observed, any error in the measurement of the base will be reproduced in the triangulation. Thus, if the measurement of the base makes it 1/500 longer than it really is, a point in the triangulation distant say, 10,000 yards from the base, will be placed 20 yards too far from it.

In all cases the accuracy with which the base is measured should be such that the errors in position of the furthest point of the triangulation, resulting from a multiplication of the initial error in the base, do not exceed the permissible maximum.

2. Bases for primary triangulation are measured with invar tapes suspended in catenary at a constant tension. The probable error of such measurements is between 1/500,000 and 1/2,000,000.

3. For artillery purposes an accuracy of 1/10,000 to 1/20,000 is all that is necessary, and this can generally be attained with steel tapes laid along the ground.

Corrections must be made for :—

(*a*). Standard.
(*b*). Temperature.
(*c*). Tension.
(*d*). Slope.
(*e*). Alignment.
(*f*). Height above the sea.
(*g*). Sag.

4. *Correction for standard.*—Owing to the tension applied in making each measurement, the length of the tape may change between the beginning and end of the operation. A *reference tape* is therefore necessary, with which the *field* tape should be checked before and after the measurement. The former should never be used for any other purpose than to check the field tape.

5. *Correction for temperature.*—The coefficient of expansion of steel is between ·00000625 and ·00000661 for each degree Fahrenheit.

6. *Tension.*—Correction for tension should be eliminated, where possible, by applying a constant tension to both field reference tapes by means of a spring balance. A tension of about 20 lbs. is sufficient for 100′ to 300′ steel tapes.

7. *Slope.*—It is very seldom that a perfectly flat site for a base can be found. All slopes in the length of a base must therefore be measured with a theodolite or level, and the measured lengths along the ground reduced to the distances along the horizontal.

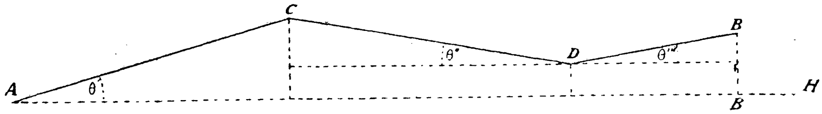


Fig. 13.

Thus in Figure 13 if A, C, D, B, is a section of the base AB, and AH a horizontal plane, then AB′, the horizontal distance between A and B, is equal to AC cos θ' + CD cos θ'' + DB cos θ''' where θ', θ'' and θ''' are the measured slopes in the length AB.

The corrections which are obviously all negative are AC (1-cos θ'), CD (1-cos θ'') and DB (1-cos θ''').

8. *Alignment.*—Corrections for alignment are only necessary when it is impossible to measure the direct line joining the two ends of the base. A deviation, if unavoidable, must be corrected for in a similar manner to slope.

9. *Height above the sea.*—All measurements are supposed to be made at sea level. In order to satisfy this condition it is necessary to reduce the base to the length it would have been if measured at sea level.

When this is done, the whole triangulation depending on it will be automatically corrected.

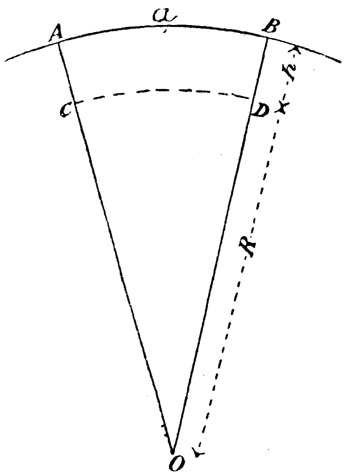


Fig. 14.

Thus in Fig. 14 if AB is the base whose measured length in feet is *a* and *h* is its mean height above sea level CD, and R the mean radius of the earth in feet, then :—

$$CD : a : : R \; R + h.$$

The correction AB — CD is

$$a\left(1 - \frac{R}{R+h}\right) = a \times \frac{h}{R+h}$$

R may be taken as 20,900,000 feet and for practical purposes this correction becomes $a \times \frac{h}{R}$, and is always negative.

10. *Sag.*—Sag should be avoided whenever possible by selecting an even site for the measurement. It may be necessary, however,

as when crossing a stream or cutting, to support the tape on pickets, or by other means, and to correct for the resulting sag as follows :—

If s = correction in feet for sag.
l = length of the tape in feet.
w = the weight of the tape in pounds.
t = the tension applied in pounds.

Then $s = l \dfrac{w^2}{24\,t^2}$ approximately, and is always negative.

19. Procedure in the Field and Subsequent Computation.

1. The ends of the base having been selected, each should be marked in a suitable manner. Mark stones cut with a circle and dot may be put into the ground, or the socket of a bearing picket (*See* Chap. XII, Section **48**) may be used. Wooden pegs driven flush with the ground may also be used for temporary work, though a more permanent mark should always be put in if it can be done.

2. The line of the base must next be cleared of obstacles, such as trees, bushes, etc.

3. A theodolite should then be set up over one mark, and aligned on a signal centred over the other mark. Intermediate marks should then be aligned along the base at intervals of one tape's length from centre to centre. These marks may be wooden pegs or the metal " arrows " used with the ordinary surveyors' chain.

4. The measuring should be done by two parties. The first consisting of three men with a steel tape and a supply of " markers," and the second, following them, consisting of two men with a theodolite and a banderole.

5. The markers should consist of small flat blocks of wood about 4 inches long, 2 inches wide and ¾ inch thick, through which are driven three long nails, and to the upper surface of which is stuck or nailed a piece of paper or white xylonite.

6. The leading tape man carries about 6 or 8 of these. The leading tape man walks along the base, and halts at the first intermediate mark. The tape is then stretched taut, a spring balance if available being used for this purpose, and aligned exactly. A " marker " is now pushed into the ground under the forward end of the tape. The rear tape man brings the end of the tape exactly over the terminal mark calling out " On " when the agreement is attained.

The third man marks the position of the other extremity of the tape on the marker with a sharp pencil, and rules a short length of line through this point at right angles to the direction of the base.

7. The tape is then lifted to one side, and drawn forward to the next intermediate mark where the procedure is repeated, the first marker being taken up by the rear tape man as soon as the second one is in position, and ruled up.

8. Whenever a change of slope occurs, the point should be marked by placing a marker in position, and noting its reading on the tape. The marker should be picked up by the theodolite party, which should follow the tape men, and measure the slopes.

9. Whenever time permits, a base should be measured at least once in each direction.

10. A base can be measured in this way with a 300 foot steel tape with a probable error of about 1/15000 (if a more accurate measurement than this is required, the method described in the Text Book of Topographical Surveying, Chapter II, should be used. This should, however, rarely be required for artillery survey purposes). It must be remembered, however, that the accordance between the various measurements, from which the probable error is deduced is, at the most, only a proof of steadiness in measuring, and is not an absolute criterion of the accuracy of the final result.

Since this result influences the whole triangulation, the greatest care must be taken in all the operations of base measurement, which should always be done in the most accurate way that time and circumstances permit.

11. The following is an example of the computation of the base measurement effected in the manner described above :—

(*a*) *Correction for standard.*—Before the first measurement the field tape was found to be ·6 inches (= ·05 feet) short of standard at 64° F. After the first and before the second measurement the F.T. was found to be ·5 inches (= ·04 feet) short of standard at 64° F. After the second measurement the F.T. was found to be ·35 inches (= ·03 feet) short of standard at 69° F. The reference tape was ·021′ long at 60° F. The coefficient of expansion given by the makers being ·00000625.

At 64° F. the reference tape was $\frac{4 \times 300 \times 6{\cdot}25}{1{,}000{,}000}$ longer than at 60° *i.e.*, ·0075 per cent. longer. It was therefore ·029′ too long.

The field tape was ·045′ short of the reference tape (*i.e.* mean of before and after comparisons).

The field tape was therefore ·016′ too short during the first measurement.

After the second measurement the reference tape was :—

$\frac{9 \times 300 \times 62{\cdot}5'}{1{,}000{,}000}$ longer than at 60° (·017 feet longer).

The reference tape was therefore ·029 feet too long before and ·038′ too long after the second measurement giving a mean ·033 too long.

The field tape was ·035 short of the reference tape and was therefore ·002 too short.

The result of the two measurements was :—

First measurement 2561·2′ feet.
Second measurement 2561·4′ feet.

The correction for standard and temperature, assuming the temperature remained between the limits stated during the measurements, is therefore,—

First measurement $+ \frac{2561 \times {\cdot}016}{300} = + {\cdot}156$

Second measurement $+ \frac{2561 \times {\cdot}002}{300} = + {\cdot}017$

(*b*) *Tension.*—No correction necessary.
(*c*) *Slope.*—(*a*) First slope 873′ at 0° 15′
correction — 873 (1-cos 0° 15′) = — ·0008
(*b*) Second slope 856′ at 1° 2′.
correction 856 (1-cos 1° 2′) = — ·1392
(*c*) Third slope 832′ at 1° 14′
correction 832 (1-cos 1° 14′). = — ·1928
Total correction for slope = — ·3228

(*d*) *Alignment.*—No correction necessary.
(*e*) *Height above sea.*—Mean height 540′

$$\text{correction} \quad -\frac{2561 \times 540}{20{,}900{,}000} = -\cdot066 \text{ nearly.}$$

	First.	Second.
Total corrections	+ ·156	+ ·017
	— ·323	— ·323
	— ·066	— ·066
	— ·233	— ·372
Actual	2561·2	2561·4
Corrected	2560·97	2561·032

Mean value 2561·0 feet.

CHAPTER VI.

COMPUTATION OF TRIANGULATION.

20. Preliminary Work, Adjustment of Figures.

1. The computation of a triangulation is done in the following stages :—

(*a*) Closing of triangles and adjustment of figures.
(*b*) Solution of triangles.
(*c*) Computation of co-ordinates of trigonometrical stations.
(*d*) Computation of length and bearing between trig points forming bases of triangles fixing intersected points, when such trig points do not form one of the sides computed in (*b*).
(*e*) Computation of the angles of the triangles fixing the intersected points.
(*f*) Solution of the triangles fixing the intersected points.
(*g*) Computation of the co-ordinates of the intersected points.
(*h*) Computation of heights of stations.
(*i*) Computation of heights of intersected points.
(*j*) Preparation of lists of co-ordinates and abstracts, etc.

2. The following checks are applicable to any system of triangulation :—

(*a*) The three angles of a plane triangle should add up to 180°. Actually a triangle on the earth's surface, as observed with a theodolite, is a spherical triangle in which the three angles are never *less* than 180°. The excess over 180° is known as spherical excess, and depends on the area of the triangle, but is so small that it can be ignored in minor and tertiary work (for example, the spherical excess of a triangle of 100 square miles area is less than 2 seconds).

(*b*) The "interior" angles of a closed polygon plus four right angles equal twice as many right angles as the figure has sides.

(*c*) In a quadrilateral or polygonal figure, such as those shown in figure 15, the product of the sines of all the "X" angles should be equal to the product of all the "Y" angles.

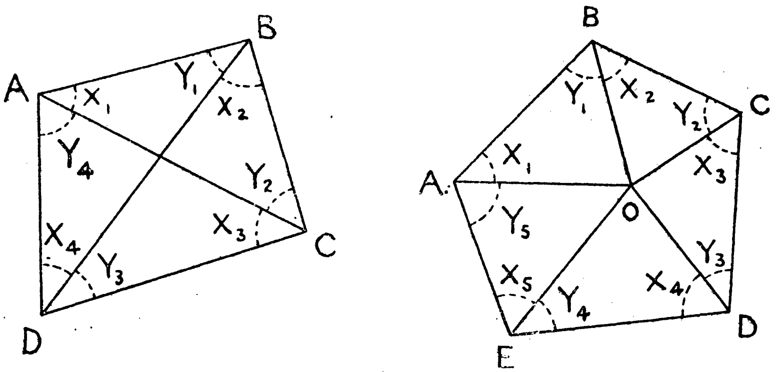


Fig. 15.

This may easily be proved thus:—

Considering the pentagonal figure:—

$$\begin{aligned} AE &= \frac{OA \sin AOE}{\sin X_5} \\ &= \frac{OB \sin Y_1}{\sin X_1} \times \frac{\sin AOE}{\sin X_5} \\ &= \frac{OC \sin Y_2 \sin Y_1}{\sin X_2 \sin X_1} \times \frac{\sin AOE}{\sin X_5} \\ &= \frac{OD \sin Y_3 \sin Y_2 \sin Y_1}{\sin X_3 \sin X_2 \sin X_1} \times \frac{\sin AOE}{\sin X_5} \\ &= \frac{OE \sin Y_4 \sin Y_3 \sin Y_2 \sin Y_1}{\sin X_4 \sin X_3 \sin X_2 \sin X_1} \times \frac{\sin AOE}{\sin X_5} \\ &= \frac{AE \sin Y_5 \sin Y_4 \sin Y_3 \sin Y_2 \sin Y_1}{\sin AOE \sin X_4 \sin X_3 \sin X_2 \sin X_1} \times \frac{\sin AOE}{\sin X_5} \end{aligned}$$

$$\therefore \frac{\sin Y_1 \sin Y_2 \sin Y_3 \sin Y_4 \sin Y_5}{\sin X_1 \sin X_2 \sin X_3 \sin X_4 \sin X_5} = 1$$

(*d*) In the case of intersected points, where these angular checks cannot be applied, each point should be fixed by two triangles having one common side.

The two values obtained from this side, one from each triangle, should agree within small limits (*i.e.*, the first four decimal places of the log value should be identical).

3. The first step in the computation of a triangulation is to close each triangle. The three observed angles of each triangle should be added up, and the difference from 180° should be noted.

If a 5-inch vernier instrument has been used for observing, and each angle measured on two zeros, a mean closing error of about 20″ may be expected. With reasonably careful observation the maximum error should not exceed 1 minute.

The magnitude of these triangular closing errors gives a useful criterion of the care and skill of the observer.

The error in each triangle should be distributed equally between the three angles of it, taking care, however, that when the angles form parts of a complete round, the complete round, after correction of the angles comprised in it, continues to add up to 360°.

4. As soon as the triangular errors have been adjusted the checks (*b*) and (*c*) should be applied, and any errors distributed throughout the work.

The check (*c*) should be done on the following form :—

Explanation of columns in adjustment of quadrilateral.

Columns 2 *and* 3.—The observed angles add up to 360° 00′ 06″. Dividing this 06″ by 8 we get ·75″. By taking off the ·75″ from each angle in Column 2 we get the angles in Column 3, which add up to 360°.

Column 4.—$X^1 + Y^1$ differ from $X_3 + Y_3$ by 08″. If we add 02″ to X_1 and Y_1 and take off 02″ from X_3 and Y_3 we make $X_1 + Y_1 = X_3 + Y_3$, whilst keeping the total of all the angles equal to 360°. Similarly for the adjustment to make $X_2 + Y_2 = X_4 + Y_4$.

Column 5.—Is the result of applying the corrections in Column 3 and 4 to Column 2.

Columns 6, 7 *and* 8.—Log sin X_1 + Log sin X_2 + &c. has to be made equal to Log sin Y_1 + Log sin Y_2 + &c. But on adding up Columns 6 and 7 we find a difference of ·0000433. Column 8 is the difference in the last figures, with their appropriate signs, of the logarithms corresponding to 1″ change in the angles in Column 5. Added together these are ·0000210. Therefore $\frac{\cdot 0000210}{8}$ is the average value of the alteration of the logarithms corresponding to an alteration of 1″ of angle. To make Column 6 equal to Column 7 we want to take off from the logarithm of *each* angle in Column 7 $\frac{\cdot 0000433}{8}$, and to add the same amount to the logarithm of each angle in Column 7. But this is equivalent to altering each *angle* by $\left(\frac{\cdot 0000433}{8} \div \frac{\cdot 0000210}{8}\right)''$, *i.e.*, by approximately 2″.

Columns 9 *and* 10.—In Column 9 this 2″ has been added or subtracted, and in Column 10 the value of increase or decrease in logarithm is obtained by multiplying the figures in Column 8 by 2 (for 2″) and giving them their appropriate sign.

Columns 11 *and* 12.—Obtained by applying the corrections in Column 9 and 10 to Column 5, 6 and 7.

[redacted]

1	2	3	4	5	6	7	8	9	10	11	12
Angle.	Observed Angle.	Correction to 360°	Correction to opposite angles.	Angles 1st Computation.	Log. Sines.		Diff. 1″	Corrections.		Corrected Angles.	Corrected Log. sines.
					X_1 X_2 X_3 X_4	Y_1 Y_2 Y_3 Y_4		Angles	Log. sines.		
	° ′ ″	″	″	° ′ ″				″		° ′ ″	
X_1	69 11 38	−0·75	+2	69 11 39	9·970 7138		8	−2	−16	69 11 37	9·970 7122
Y_1	36 01 32	−0·75	+2	36 01 33		9·769 4881	29	+2	+58	36 01 35	9·769 4939
X_2	27 29 43	−0·75	+0·5	27 29 43	9·664 3368		40	−2	−80	27 29 41	9·664 3288
2	47 17 05	−0·75	+0·5	47 17 05		9·866 1300	19	+2	+38	47 17 07	9·866 1338
				180 00 00							
X_3	77 52 12	−0·75	−2	77 52 09	9·990 1924		5	−2	−10	77 52 07	9·990 1914
Y_3	27 21 06	−0·75	−2	27 21 03		9·662 2267	41	+2	+82	27 21 05	9·662 2349
X_4	22 00 24	−0·75	−0·5	22 00 23	9·573 6953		52	−2	−104	22 00 21	9·573 6849
Y_4	52 46 26	−0·75	−0·5	52 46 25		9·901 0502	16	+2	+32	52 46 27	9·901 0534
	360 00 06			180 00 00	9·198 9383	9·198 8950	210				
	8)06				8950						
	0·75				433						

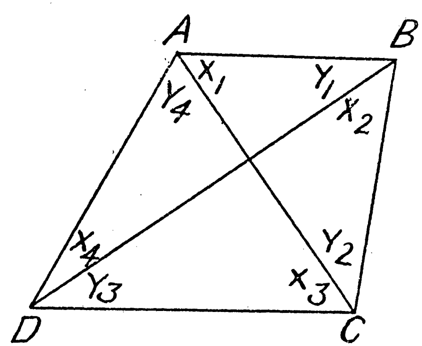
13		14	
Computation of sides.		Log. lengths of sides.	
Log. Sin. X_1	9·970 7122	B.C.	3·894 2326
A.B.	3·789 6542		
Cosec Y_2	0·133 8662	A.C.	3·875 3913
Sin.(X_2+Y_1)	9·951 8709		
Sin. X_2	9·664 3288	C.D.	3·896 3265
B.C.	3·894 2326		
Cosec Y_3	0·337 7651	B.D.	4·144 5431
Sin.(X_3+Y_2)	9·912 5454		
Sin. X_3	9·990 1914	D.A.	3·985 4645
C.D.	3·896 3265		
Cosec Y_4	0·098 9456	C.A.	3·875 3920
Sin.(X_4+Y_3)	9·880 1189		
Sin. X_4	9·573 6849	A.B.	3·789 6555
D.A.	3·985 4645		
Cosec Y_1	0·230 5061	D.B.	4·144 5436
Sin.(X_1+Y_4)	9·928 5730		

$(X_1 + Y_1) = 105°\ 13'\ 10''$
$(X_3 + Y_3) = 105°\ 13'\ 18''$

4)08
02

$(X_2 + Y_2) = 74°\ 46'\ 48''$
$(X_4 + Y_4) = 74°\ 46'\ 50''$

4)02
00·5



	°	′	″
$(X_2 + Y_1)$	63	31	16
$(X_3 + Y_2)$ =	125	09	14
$(X_4 + Y_3)$ =	49	21	26
$(X_1 + Y_4)$ =	121	58	04

Values of A.B. = 6161.04
6161·06

Diff. = 0·02′ = 0·24″

Explanation of columns in adjustment of polygon.

Columns 2 *and* 3.—In the case of the angles X_1 Y_1 and Z_1, since these add up to 180° 00′ 04″, we have to subtract 4″ from their total to make their sum equal to 180°. This is done by subtracting 1″ from X_1, 1″ from Y_1 and 2″ from Z_1. Similarly for the other triangles.

Column 4.—The central angles have already been adjusted by the values in Column 3. *Taking these into account* the sum of the central angles adds up to 360° 00′ 02″. These are adjusted by taking 1″ off (say) Z_2 and Z_4. To compensate for this, in order to keep the three angles of each triangle equal to 180°, 1″ must be added to Y_2 and Y_4.

Column 5.—Is the result of applying the corrections in Column 3 and 4 to Column 2.

Column 6, 7 *and* 8.—Log sin X_1 + Log sin X_2, &c., has to be made equal to Log sin Y_1 + Log sin Y_2 + &c. But in adding up Columns 6 and 7 there is a difference of ·0000559. Column 8 is the difference in the last figures, with their appropriate signs, of the logarithms corresponding to 1″ change in the angles in Column 5. Added together these are equal to ·0000157. Therefore $\frac{\cdot 0000157}{12}$ is the average value of the alteration of the logarithms corresponding to an alteration of 1″ of angle. To make Column 6 equal to Column 7 $\frac{\cdot 0000559}{12}$ must be added to the logarithm of each angle in Column 6 and the same amount deducted from the logarithm of each angle in Column 7. But this is equivalent to altering each angle by $\left(\frac{\cdot 0000559}{12} \div \frac{\cdot 0000157}{12}\right)''$—*i.e.*, by approximately 3·5″.

Columns 9 *and* 10.—In Column 9 this 3·5″ (4″ to one-half the angle and 3″ to the other half) has been added or subtracted, and in Column 10 the value of the increase and decrease in logs is obtained by multiplying the figures in Column 8 by the figures in Column 9 and giving them their appropriate signs.

Columns 11 *and* 12.—Obtained by applying the corrections in Columns 9 and 10 to Columns 5, 6 and 7.

5. In addition to these checks, which may be called "internal" checks, a system of triangulation may be checked in the following ways :—

(*a*) The triangulation, if based on the sides of a triangulation of a higher order of accuracy, may be made to include and close on one or more additional points of this latter triangulation.

(*b*) By measurement of check bases at one or more points. This gives a check on the computed size of the triangles.

Adjustment of Polygon.

1	2	3		4	5	6	7	8	9	10	11	12				
Angle.	Observed Angle.	Correction to 180°	Central Angles.	Correction 360°	Angles 1st Computation.	Log. Sines. X Angles.	Log. Sines. Y Angles.	Diff. 1″	Corrections. Angles	Corrections. Log. Sines.	Corrected Angles.	Corrected Log. Sines.	Computation of Sides.		Log. length of sides.	
	° ′ ″				° ′ ″						° ′ ″					
X_1	40 44 14	−1			40 44 13	9·814 6385		+25	+4	+100	40 44 17	9·814 6485	Sin. X_1	9·814 6485	A.G.	3·422 0784
Y_1	49 58 36	−1		+1	49 58 36		9·884 1055	+18	−4	− 72	49 58 32	9·884 0983	Dis. A.B.	3·607 3962		
Z_1	89 17 14	−2	12″	−1	89 17 11						89 17 11	9·999 9663	Cosec Z_1	0·000 0337		
	180 00 04	−4			180 00 00								Sin. Y_1	9·884 0983	B.G.	3·491 5282
X_2	88 54 31	+3			88 54 34	9·999 9213		+1	+3	+ 3	88 54 37	9·999 9216	Sin. X_2	9·999 9216	F.G.	3·536 2858
Y_2	50 13 50	+3			50 13 53		9·885 7196	+18	−3	−34	50 13 50	9·885 7142	A.G.	3·422 0784		
Z_2	40 51 31	+2	33″		40 51 33						40 51 33	9·815 7118	Cosec Y_2	0·114 2858		
	179 59 52	+8			180 00 00								Sin. Z_2	9·815 7118	A.F.	3·352 0760
X_3	55 00 15	+1			55 00 16	9·913 3881		+15	+4	+ 60	55 00 20	9·913 3941	Sin. X_3	9·913 3941		
Y_3	67 51 30	+2			67 51 32		9·966 7322	+8	−4	− 32	67 51 28	9·966 7290	F. G.	3·536 2858	E.G.	3·482 9509
Z_3	57 08 10	+2	12″		57 08 12						57 08 12	9·924 2624	Cosec Y_3	0·033 2710		
	179 59 55	+5			180 00 00								Sin. Z_3	9·924 2624	E.F.	3·493 8192
X_4	51 58 49	−1			51 58 48	9·896 4137		+16	+3	+ 48	51 58 51	9·896 4185	Sin. X_4	9·896 4185	G.D.	3·526 6827
Y_4	45 25 35	−1		+1	45 25 35		9·852 6930	+21	−3	− 63	45 25 32	9·852 6867	E.G.	3·482 9509		
Z_4	82 35 39	−1	38″	−1	82 35 37						82 35 37	9·996 3614	Cosec Y_4	0·147 3133		
	180 00 03	−3			180 00 00								Sin. Z_4	9·996 3614	E.D.	3·626 6256
X_5	71 40 47	−3			71 40 44	9·977 4079		+7	+3	+ 21	71 40 47	9·977 4100	Sin. X_5	9·977 4100		
Y_5	68 45 15	−3			68 45 12		9·969 4294	+8	−3	− 24	68 45 09	9·969 4270	D.G.	3·526 6827	C.G.	3·534 6657
Z_5	39 34 06	−2	04″		39 34 04						39 34 04	9·804 1330	Cosec Y_5	0·030 5730		
	180 00 08	−8			180 00 00								Sin. Z_5	9·804 1330	C.D.	3·361 3887
X_6	58 43 16	+3			58 43 19	9·931 7922		+13	+4	+ 52	58 43 23	9·931 7974	Sin. X_6	9·931 7974		
Y_6	70 43 14	+4			70 43 18		9·974 9379	+7	−4	− 28	70 43 14	9·974 9351	C.G.	3·534 6657	B.G.	3·491 5280
Z_6	50 33 20	+3	23″		50 33 23						50 33 23	9·887 7580	Cosec Y_6	0·025 0649		
	179 59 50	+10			180 00 00								Sin. Z_6	9·887 7580	B.C.	3·447 4886
	Sum of Central Angles = 360° 00′ 02″					9·533 5617	9·533 6176 5617	157								
							157) 559(	=3·5	Total 42·0							

(*c*) By astronomical observation. Astronomical observation is particularly valuable for controlling errors of direction or orientation. The probable error in astronomical determinations of *position*, even by the most accurate known methods, is of the order of 20 or 30 yards. They are, therefore, only useful for detecting large accumulations of error such as may be expected in an extended series.

They are unlikely, therefore, to be required by the artillery surveyor.

21. *Solution of Triangles.*

1. Having adjusted the angles of the various triangles, the lengths of the sides of each triangle are computed from the formulæ :—

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Thus, if the side CB (fig. 16) is the base of the triangle ABC

$$b = a \sin B \operatorname{cosec} A$$
$$c = a \sin C \operatorname{cosec} A$$

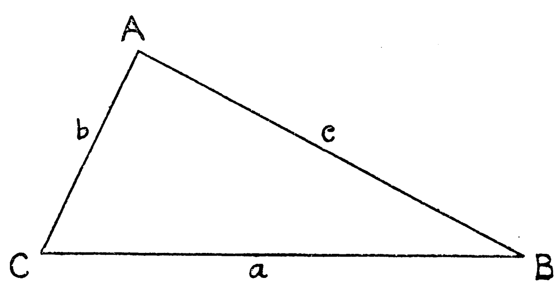


FIG. 16.

For logarithmic computation it is convenient to write these elements down in a definite order, thus :—

$$\left.\begin{array}{l}\log \sin B \\ \log a \\ \log \operatorname{cosec} A\end{array}\right\} \cdots \cdots \text{ adding first three gives } \log b$$

$$\left.\begin{array}{l}\log a \\ \log \operatorname{cosec} A \\ \log \sin C\end{array}\right\} \cdots \cdots \text{ adding last three gives } \log c$$

2. The form for the solution of the triangles of an adusted polygon can be made an extension of the form required for making the adjustment, thus obviating the necessity of looking out the logarithms of the sines a second time.

3. It is occasionally necessary to compute the length of the side of a triangle from the lengths of the other two sides and the included

angle between them. In this case it is necessary first to compute the other two angles from the formulæ—

$$\tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}$$

$$\frac{A + B}{2} = 90° - \frac{C}{2}.$$

Having found A and B thus, the triangle can be solved by the formulæ given in para. 1.

22. Computation of Co-ordinates.

1. Computation of the co-ordinates of one point from another is done from the formulæ—

$$x = x_1 + l \sin \theta$$
$$y = y_1 + l \cos \theta,$$

where x and y are the co-ordinates to be computed,

x_1 and y_1 are the co-ordinates of the known point,
l the distance between the two points,
θ the bearing from the known to the unknown point.

2. It is first necessary to deduce the mutual bearings of the various stations from the known bearing of the base by adding or subtracting, as the case may be, the corrected observed angles.

Thus, in Fig. 17, if AB is the base, then the bearing AC will be the bearing AB less the observed angle CAB.

The bearing CA will be 180° + bearing AC, and the bearing CD will be the bearing CA less the observed angle ACD.

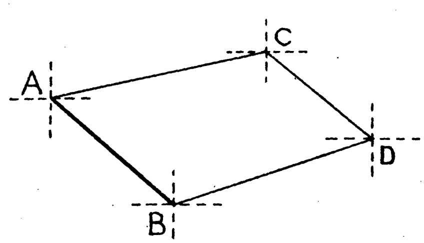


Fig. 17.

3. The computation is really effected in two stages, the values of $l \sin \theta$ and $l \cos \theta$ being computed with the aid of logarithms, and the values so found added algebraically to x_1 and y_1 respectively, giving each its proper sign according to the signs of $\sin \theta$ and $\cos \theta$.

Thus in computing C from A, the bearing is in the first quadrant and the signs of both are positive. The value found for $l \sin \theta$ and $l \cos \theta$ must be added to x_1 and y_1. In computing D from C the bearing is in the second quadrant. The $\sin \theta$ is therefore positive and $\cos \theta$ negative, so that the value of $l \sin \theta$ must be added to x_1 and $l \cos \theta$ subtracted from y_1.

This is apparent from the figure, and it is always advisable to draw out a rough diagram of the triangulation to ensure that the signs are inserted correctly, and that no mistake is made by adding when subtraction ought to be done, and *vice versâ*.

23. Computation of Heights.

1. When the triangulation is based on the side of some pre-existing triangulation, two initial heights are available. When a base has to be measured the initial height of one end must be determined by some independent means, *e.g.*, careful barometer readings at the point and at some other point whose height above datum is known.

2. All vertical angles require correction for curvature of the earth and terrestrial refraction.

3. *Curvature.*—The curvature of the earth makes an object appear lower than it really is.

In Fig. 18, if C represents the centre of the earth, A the observer, and B another point on the same level as A, and if AP is a tangent at A, then the vertical angle observed to P at A will be zero. P will appear to be on the same level as A, and will be placed too low by the distance x.

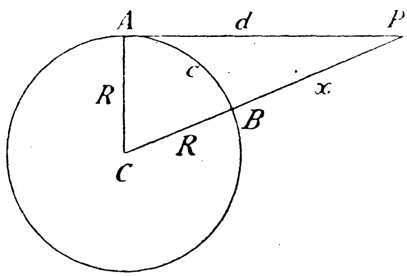


Fig. 18.

Then $(R + x)^2 = R^2 + d^2$ or $x\,(2R + x) = d^2$.

Since x is very small compared with 2R, and since for short distances AP = AB approximately, we have $x = \dfrac{C^2}{2R}$.

The mean diameter of the earth may be taken as 7,916 miles, whence x at 1 mile = about 8 inches, 2 miles = 32 inches, and so on, increasing as the square of the distance.

The simple formula for curvature is, therefore, "the correction for curvature in *feet* is two-thirds of the square of the distance in *miles*."

4. *Refraction.*—Owing to the change of density of the air as height above ground increases, a ray to a distant object does not pass through

a homogeneous medium and is bent or refracted, making the object appear higher than it really is.

This can be seen from Fig. 19, where A represents an observer, and B the object observed.

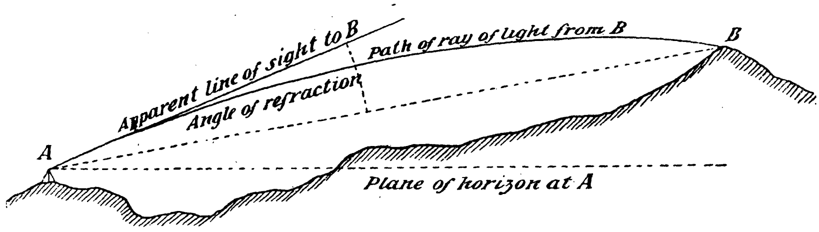


FIG. 19.

The amount of the deflection, or angle of refraction, varies with the place, season and time of day.

5. Terrestrial refraction must not be confused with astronomical refraction, which is concerned with angles of elevation of 10° or more and of which the laws are fairly well known.

Terrestrial refraction is very variable, and should whenever possible, be calculated from local and contemporary observations.

Suppose the calculated mean refraction be r, then $\frac{r}{c}$, where c is the distance AB, is called the " co-efficient of refraction." This varies in different countries. The mean value may be taken as about 0·07.

The correction, y, for terrestial refraction may be taken as $y = \frac{kc^2}{R}$ where k is the co-efficient of refraction.

6. This correction may conveniently be combined with the correction for curvature, the combined correction being given by $y = \frac{c^2}{R}\left(\frac{1-2k}{2}\right)$, or stated as a rule of thumb, taking k as 0·07.

The combined correction for curvature and refraction in feet is four-sevenths of the square of the distance in miles.

This is only an approximate rule, and should only be used when reciprocal observations are impossible.

7. In Fig. 20, if A and B be two trig stations, R and R^1 the apparent (refracted) rays to the opposite station.

It is assumed that $ABR^1 = BAR$. Make $OC = OA$. It is required to find the length BC.

Let Da and Db be the observed depressions at A and B respectively. Then—

$$BAC = OCA - OBA,$$

and $BAC = BAO - CAO$, and since $OCA = CAO$,

$$BAC = \tfrac{1}{2}(BAO - ABO) = \tfrac{1}{2}(RAO - R^1BO) = \tfrac{1}{2}(Db - Da).$$

8. *Corrections for heights of beacons and instruments.*—Thus far it has been assumed that the observations have been made from the ground level at each station to ground level at the other.

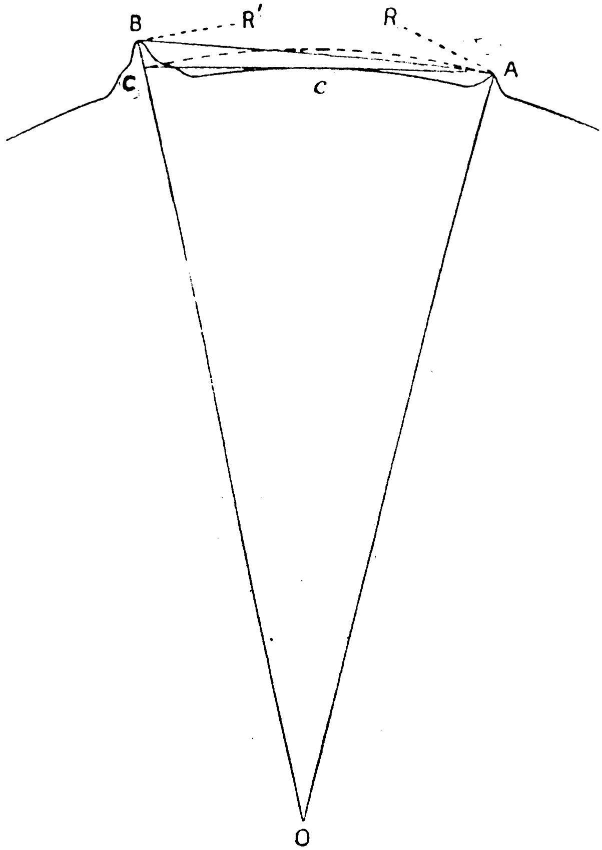


FIG. 20.

If i_a be the height above ground level at A of the instrument at A, which is being used to observe to a beacon of height g_b above ground level at B.

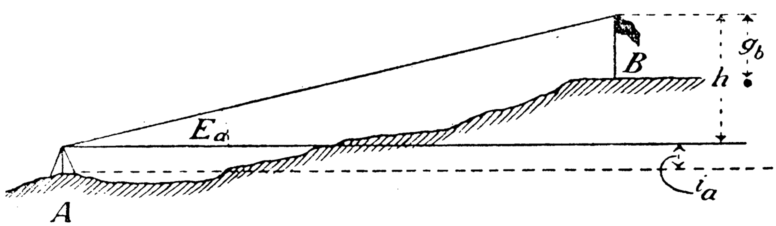


FIG. 21.

Then from Fig. 21—
If h be the height of the beacon above the instrument, the point B is above the point A by

$$h - g_b + i_a$$

Generally, in reciprocal observations if i_a, i_b, be the heights of the instruments above ground level at A and B, and g_a, g_b, the heights of the beacons above ground level at A and B to which observations have been taken, the corrections to be applied to the already calculated difference of height is—

$$\frac{+g_a - g_b + i_a - i_b}{2}.$$

Examples of computation of trigonometrical heights :—

D and E_a denote vertical angle of depression or elevation at A.

D_b and E_b denote vertical angle of depression or elevation at B.

i_a and g_a denote height of instrument and beacon above ground level at A.

i_b and g_b denote height of instrument and beacon above ground level at B.

Case I.—If reciprocal vertical angles have been observed at A and B use space 1 (*a*) and omit 1 (*b*). In this case angle $S = \frac{D_b - D_a}{2}$, but should the angle at either A or B be elevation, its sign in the foregoing expression must be changed.

The correction for height of instrument and beacons is—

$$\frac{g_a + i_a - g_b - i_b}{2}$$

Case II.—If the vertical angle at only one station has been observed use space I (*b*) and omit I (*a*). The angle S = K − D or else K + E when K is the correction for curvature and refraction obtained from the accompanying table.

If observations have been taken from A (the station of which height is known), correction for height of instrument and beacon $= + i_a - g_b$: or if the ground line at B has been taken $+ i_a$.

If observations have been taken from B (the station of which height is known), correction for height of instrument and beacon $= - i_b + g_a$.

TABLE FOR K.

Distance between Stations in Metres.	K ′	″
500	0	07
1,000	0	14
1,500	0	20
2,000	0	28
2,500	0	35
3,000	0	42
3,500	0	49
4,000	0	56
4,500	1	03
5,000	1	10
6,000	1	24
7,000	1	38
8,000	1	52
9,000	2	06
10,000	2	20
20,000	4	39
30,000	6	58

Data.			
	Station A.		
	Known height of A.	367·2 metres.	453·0 metres.
	Station B.		
	Log AB (c) in metres.	4·9901924	4·7277039
I (a)		° ′ ″	
	$+E_a$ or $-D_a$ ... ... =	− 0 6 37·3	
	$+D_b$ or $-E_b$ ... ... =	+ 0 7 24·0	
	Divide algebraic sum by	2)0 0 46·7	
	∴ S (±)	+ 0 0 23·35	
I (b)			° ′ ″
	+ E or − D ... ... =		(Ea) + 0 5 10
	K ... ... =		+ 0 12 28
	∴ S (±) ... ... =		+ 0 17 38
II			
	log tan S ... ... =	$\bar{4}$·0537719	$\bar{3}$·7100643
	log c ... ... =	4·9901924	4·7277039
	∴ log h (sum) ... ... =	1·0439643	2·4377682
	Hence h (±) ... ... =	+11·07 metres	+274·01 metres
	Correction for heights of beacon and instruments ... ... =	− 0·77 metres	+ 4·50 metres
	Known height of A ... ... =	367·2 metres	453·00 metres
	Required height of B ... =	377·50 metres	731·51 metres

24. Organization of Computing.

1. The computation of a triangulation cannot as a rule be started until the observation is well advanced, and is very commonly the slowest part of the work, unless several computers are available and the work can be divided up and distributed among them.

By careful organisation of this distribution of work time can be saved in two ways :—

(*a*) If the work can be so organised that errors can be at once detected, the time otherwise spent in searching for mistakes will be saved.

(*b*) The amount of work to be done by each man will be reduced to a minimum.

2. Detection of errors is best insured by carrying out all computations in the same form, and by having each computation done by two men, working concurrently, but separately and independently.

To do this throughout the whole work would, however, mean that each portion of it is done twice, and it is generally sufficient to duplicate the computation in this way only as far as the computation of the bases and angles of the intersected point triangles (stage (*e*) in Sect. 22 para. 2).

From this point onwards, *i.e.*, the computation of the intersected points, the internal checks should be sufficient to show if any errors have been made, and, as these must have occurred in the solution of the triangles or computation of the co-ordinates, it should not take very long to find them.

In order to provide these checks, however, each intersected point *must* be fixed by at least two triangles, and its co-ordinates and height computed from at least two points.

It is not as a rule necessary to solve more than two triangles to fix a point but the solution of a third triangle, if a third is available, is often the quickest way of finding the triangle in which a mistake has been made.

3. The exact manner in which the work is distributed will depend on the number of computers available and the number of triangles to be solved.

The first thing to decide is the order in which the triangles will be solved after the figures have been adjusted.

A triangle cannot be solved until the length of the base is known. The whole system of triangles should therefore be divided up into sections, as far as the figures permit of this being done. A decision should be made as to which sides of one section are to serve as bases for the next and all the available computers, working in pairs, concentrated successively on each section, each pair doing the same computation in the same way, but independently, and checking the computations of one man against the other at the end of the section.

4. When the lengths of all the sides have been computed the computation of the co-ordinates should be arranged so that the first few points computed will serve as starting points for the computation of as many other points as possible.

In a polygon, for example, the co-ordinates of the central point should be got out first, since, once this has been done, the computations for all the other points can proceed simultaneously; whereas if one of the outer points is taken first, only one or at most two other points can be done from it.

5. As soon as all co-ordinates have been computed an abstract of the results should be made showing :—

(*a*) the co-ordinates of each point;
(*b*) the bearings of the various points from one another;
(*c*) the logarithms of the various sides.

This is conveniently prepared in the form given below. Each pair of computers as they complete their computations should enter up the results on the abstract, one man writing in the figures and the other checking them.

In the first instance, the figures entered on this form will be derived direct from the solution of the triangles.

It will probably be necessary, however, to compute the length and bearing between some pairs of points, not belonging to the same triangle, to serve as bases for the fixing of intersected points. This must be done from their co-ordinates, and the next step is therefore to decide on the triangles of the intersected points. A list should be made out showing all the triangles to be solved and the base of each, after which any bases over and above those already determined by the solution of the triangles, formed by the stations must be computed from co-ordinates and entered upon the abstract.

7. When this has been done the computation of the intersected points, including solution of triangles, computation of co-ordinates and heights, can be divided up among the available computers, Each man being responsible for the co-ordinates and heights of the points he has to compute.

8. Finally, a list of co-ordinates of all points fixed, both stations and intersected points, must be prepared and very carefully checked. It is often desirable to prepare a chart of the whole system. This is very useful if any extension or amplification of the work is ever required as it shows at a glance which of the stations are inter-visible.

For the organisation and arrangement of the computation the reconnaissance chart is quite sufficient, but this will seldom be accurate enough to serve as the final chart which is kept as a record. This should be prepared by plotting all points from their calculated co-ordinates on a suitable scale.

9. It may be of assistance in organising the rapid computation of a triangulation if some idea is given as to the number of computers that can be efficiently employed. Much will, of course, depend on the skill of the individual man, so that only a rough idea, assuming average and equal skill in all, can be given.

10. The following principles, however, can be stated :—

(*a*) Accuracy and speedy detection of mistakes are best ensured by two men doing the same computation separately.

(*b*) It is more efficient for the computers to work separately than in pairs. For example, four computers working separately will complete a given number of computations in about two-thirds of the time taken by two pairs.

(*c*) One man should be detailed to supervise the collection of results and preparation of abstracts.

(*d*) Four triangles can be solved in the time required to determine the height of one point or its co-ordinates.

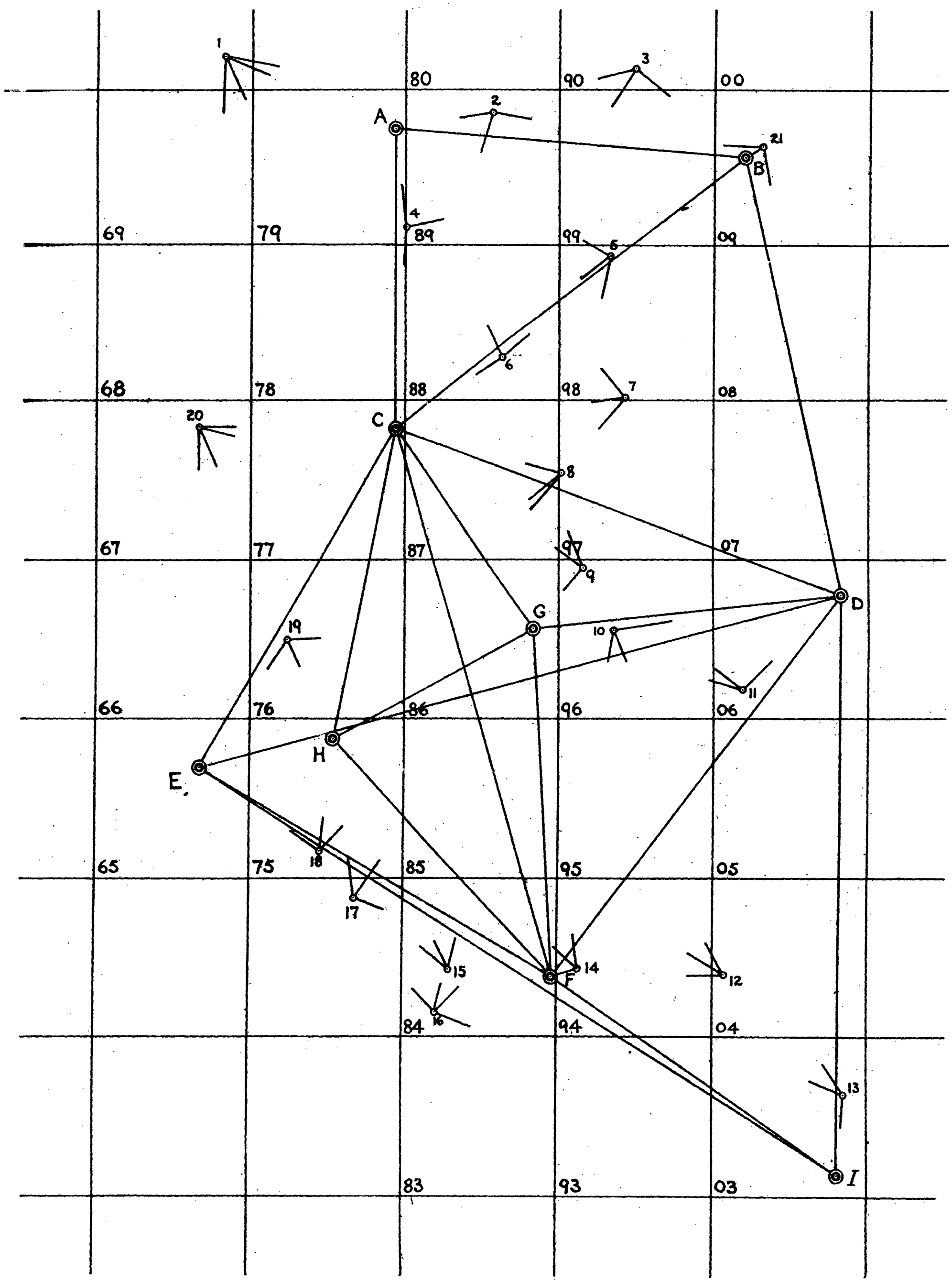
(*e*) Duplicate copies of abstracts are of great assistance in speeding up the work. All copies must, however, be most carefully checked. Mistakes in copying down data are often difficult to detect, and render *all* subsequent work useless.

11. The following is an example of the computation of a triangulation and its organisation.

Fig. 22 shows the diagram of the complete triangulation, which is based on the side EF, and consists of three quadrilateral figures and two triangles having a common side.

To face page 56.

Fig. 22.



Malby & Sons, Lith.

The various steps in the computation are :—

(*a*) The preparation of abstracts of the observations at the various stations. Those of the stations E, F, D, G and I are given as examples.

(*b*) Adjustment and solution of the quadrilaterals. The solution of the quadrilateral CDEF is given as an example.

(*c*) Solution of the triangles EDI and DFI. These are given in full and shows the sort of accordance which may be expected in the two values obtained for the common side DI, viz., 3·5558771 and 3·5558551.

(*d*) Computation of the co-ordinates of trig stations. Examples of these are included in (3).

(*e*) Preparation of the abstract of trig stations. This is commenced at this stage and completed as the computations proceed.

(*f*) Selections of triangles for the intersected points. The list given should be examined in conjunction with Fig. 22. It will be noticed that there is frequently a choice of two or possibly more, triangles for fixing any point, and that the best conditioned triangles only have been made use of.

From this list it is seen that it will be necessary to compute the bearings and distances of EB, AH, EG and EH from the co-ordinates of A, B, E, G and H already entered on the abstract (5).

(7) Computation of length and bearing of EB, AH, EG and EH. No example is given of this as it is done on the form shown on p. 12.

The results of the computation are entered up in the abstract of trig stations.

(8) Solution of triangles for intersected points.

Two examples are given, *e.g.* :—

Three triangles fixing the point 10.
Two triangles fixing the point 16.

Note in each case the accordance between the various values obtained for the length of the common sides.

(9) Computation of the co-ordinates of intersected points. An example is given of the computation of co-ordinates of the point 10.

(10) Computation of heights of trig stations. An example is given of the computation of the height of I. The stations would be taken in the following order :—

Starting from E and F two values are obtained for the height of C. The mean is accepted as correct and from E, F and C three values are obtained of the height of D.

Starting from C and D the quadrilateral ABCD is dealt with in the same way.

The height of G is fixed from C, D and F, that of H from C, F and G, and that of I from E, D and F.

(11) Computation of heights of intersected points. An example is given of the computation of the height of the point 10 from D and F.

(12) Preparation of the final list of trig points.

ABSTRACT OF ANGLES.

Inst. ...	... *603.*	Book No.	... *6478.*
Station	... *E.*	Date ...	... *29.8.22.*
Observer	... *Lieut. J. Brown.*	*Height of Instrument*	*1·4 metres.*
Booker	... *Br. H. Robinson.*	*Height of Beacon* ...	*4·8 metres.*
Face, Swing, Arc.			

——	*Horizontal.*			——	*Vertical.*			Remarks.
	°	′	″		°	′	″	
Stations—								
C	*0*	*0*	*0*	*D*	*0*	*27*	*50*	*To Cross Piece.*
D	*44*	*12*	*50*	*D*	*0*	*21*	*31*	*Top of Flag.*
F	*88*	*47*	*46*	*D*	*0*	*42*	*12*	*Top of Flag.*
I	*90*	*16*	*04*	*D*	*0*	*19*	*47*	*Ground.*
Intersected Points—	*(same zero)*							
19 ...	*03*	*11*	*10*					
8 ...	*20*	*51*	*12*	*D*	*0*	*46*	*20*	*Top.*
7 ...	*27*	*39*	*22*	*D*	*0*	*40*	*20*	*Top.*
18 ...	*92*	*10*	*02*	*D*	*0*	*27*	*03*	*Top of Cradle.*
16 ...	*104*	*04*	*18*	*D*	*0*	*14*	*20*	*Top of Tower.*

ABSTRACT OF ANGLES.

Inst ...	... *603.*	Book No. ...	... *6478.*
Station	... *F.*	Date	... *28.8.22.*
Observer	... *Lieut. J. Brown.*	*Height of Instrument*	*1·45 metres.*
Booker	... *Gnr. G. Williams.*	*Height of Beacon* ...	*6·6 metres.*
Face, Swing, Arc.			

——	*Horizontal.*			——	*Vertical.*			Remarks.
	°	′	″		°	′	″	
Stations—								
D	*0*	*0*	*0*	*E*	*0*	*13*	*50*	*Top of Flag.*
I	*85*	*31*	*24*	*E*	*0*	*08*	*10*	*Ground line.*
E	*262*	*15*	*09*	*E*	*0*	*51*	*52*	*Top of Flag.*
H	*278*	*38*	*35*	*E*	*0*	*38*	*35*	*Ground.*
C	*305*	*24*	*51*	*E*	*0*	*15*	*45*	*Cross Bar.*
G	*318*	*54*	*22*	*E*	*0*	*13*	*00*	*Ground.*
Intersected Points—	*(same zero)*							
14 ...	*33*	*04*	*09*					
12 ...	*51*	*03*	*22*	*D*	*0*	*31*	*05*	*Top of Pole.*
13 ...	*72*	*09*	*46*	*D*	*0*	*42*	*10*	*Ground.*
17 ...	*251*	*25*	*20*					
18 ...	*260*	*25*	*09*	*E*	*0*	*54*	*45*	*Top of Gable.*
7 ...	*331*	*45*	*04*	*D*	*0*	*19*	*40*	*?*
10 ...	*332*	*35*	*04*	*D*	*0*	*15*	*20*	*Top of Chimney.*
20 ...								*Not visible, haze.*

Abstract of Angles.

Inst. ...	... *598.*	Book No. ...	... *35445.*
Station	... *D.*	Date	... *28.8.22.*
Observer	... *Capt. P. Smith.*	*Height of Instrument*	*1·5 metres.*
Booker	... *Br. H. Robinson.*	*Height of Beacon* ...	*6·2 metres.*

Face, Swing, Arc.

——	*Horizontal.*			——	*Vertical.*			Remarks.
	°	′	″		°	′	″	
Stations—								
C	*0*	*0*	*0*	*E*	*0*	*10*	*39*	*Cross Bar.*
A	*26*	*48*	*36*	*E*	*0*	*04*	*28*	*Top of Flag.*
B	*58*	*19*	*22*	*E*	*0*	*18*	*19*	*Top of Flag.*
I	*251*	*25*	*19*	*E*	*0*	*03*	*20*	*Top of Flag.*
F	*288*	*37*	*16*	*D*	*0*	*03*	*40*	*Top of Flag.*
H	*325*	*48*	*12*	*E*	*0*	*17*	*01*	*Top of Flag.*
E	*326*	*18*	*16*	*E*	*0*	*23*	*25*	*Top of Tripod.*
G	*334*	*46*	*38*	*D*	*0*	*02*	*36*	*Top of Flag.*
Intersected Points—								
11 ...	*291*	*21*	*15*	*D*	*1*	*05*	*12*	*Top.*
16 ...	*296*	*04*	*12*	*E*	*0*	*17*	*55*	*Top of Tower.*
10 ...	*332*	*28*	*22*	*D*	*0*	*41*	*01*	*Top of Chimney.*

Abstract of Angles.

Inst. ...	... *596.*	Book No. ...	... *1614.*
Station	... *G.*	Date	... *29.8.22.*
Observer	... *Capt. P. Smith.*	*Height of Instrument*	*1·5 metres.*
Booker	... *Br. H. Robinson.*	*Height of Flag* ...	*2·8 metres.*

Face, Swing, Arc.

——	*Horizontal.*			——	*Vertical.*			Remarks.
	°	′	″		°	′	″	
Stations—								
F	*0*	*0*	*0*	*D*	*0*	*06*	*10*	*Top of Tripod.*
H	*66*	*04*	*57*	*E*	*0*	*38*	*15*	*Ground.*
C	*146*	*43*	*51*	*E*	*0*	*24*	*20*	*Cross Piece.*
D	*267*	*14*	*15*	*E*	*0*	*09*	*20*	*Top of Flag.*
Intersected Points—	*(same zero)*							
15 ...	*17*	*34*	*21*					
16 ...	*18*	*20*	*40*					
17 ...	*38*	*32*	*46*	*E*	*0*	*25*	*21*	*Top.*
18 ...	*48*	*32*	*17*	*E*	*0*	*40*	*25*	*North Gable Top.*
19 ...	*89*	*12*	*04*	*E*	*0*	*11*	*32*	*Top.*
7 ...	*195*	*13*	*07*	*D*	*0*	*04*	*15*	*Top of Chimney.*
8 ...	*195*	*20*	*32*	*D*	*0*	*33*	*25*	*Top.*
9 ...	*243*	*15*	*45*	*D*	*0*	*01*	*16*	*Top of Chimney.*
11 ...	*289*	*26*	*11*	*D*	*0*	*11*	*21*	*Top.*
12 ...	*333*	*18*	*10*	*D*	*0*	*35*	*05*	*Top of Flagstaff.*
14 ...	*355*	*31*	*01*	*D*	*0*	*07*	*11*	*Top.*

ABSTRACT OF ANGLES.

Inst. ... ... *598.*	Book No. ... ... *35445.*
Station ... *I.*	Date ... ... *25.8.22.*
Observer ... *Capt. P. Smith.*	*Height of Instrument* *1·65 metres.*
Booker ... *Lieut. J. Jones.*	*Height of Beacon* ... *5·4 metres.*
Face, Swing, Arc.	

——	*Horizontal.*			——	*Vertical.*			Remarks.
	°	′	″		°	′	″	
Stations—								
F	*0*	*0*	*0*	*D*	*0*	*07*	*55*	*Top of Tripod.*
D	*57*	*16*	*32*	*E*	*0*	*04*	*20*	*Ground Line.*
E	*358*	*12*	*35*	*E*	*0*	*21*	*17*	*Top of Tripod.*
Intersected Points—	*(Same zero)*							
10 ...	*33*	*52*	*09*	*D*	*0*	*16*	*16*	*Top of Chimney.*
13 ...	*58*	*31*	*20*	*D*	*1*	*02*	*15*	*Top.*
16 ...	*347*	*30*	*59*	*E*	*0*	*26*	*00*	*Top of Tower.*

ADJUSTMENT OF QUADRILATERAL.

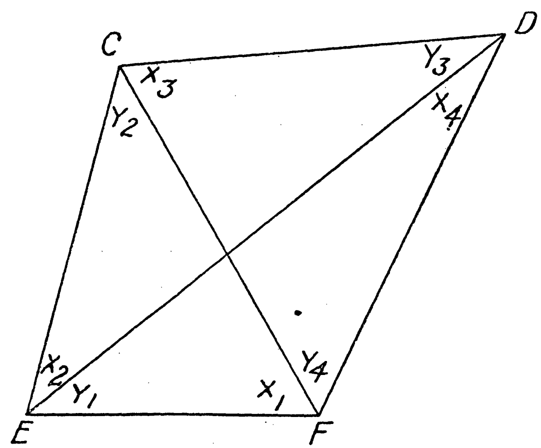
Angle.	Observed Angle. ° ′ ″	Correction to 360° ″	Correction to opposite angles. ″	Angles 1st Computation. ° ′ ″	Log Sines. 1 3, 5, 7.	Log Sines. 2, 4, 6, 8.	Diff. 1″	Corrections. Angles ″	Corrections. Log sines.	Corrected Angles. ° ′ ″	Corrected Log. sines.
X_1	43 09 42	−7	−2	43 09 33	9·8350735		+23	−2	−47	43 09 31	9·8350688
Y_1	44 34 56	−7	−2	44 34 47		9·8462759	+21	+2	+43	44 34 49	9·8462802
X_2	44 12 50	−6	+8	44 12 52	9·8434482		+22	−2	−45	44 12 50	9·8434437
Y_2	48 02 48	−7	+7	48 02 48		9·8713917	+19	+2	+39	48 02 50	9·8713956
				180 00 00							
X_3	54 02 46	−7	+2	54 02 41	9·9082037		+16	−2	−33	54 02 39	9·9082004
Y_3	33 41 44	−7	+2	33 41 39		9·7441049	+32	+2	+65	33 41 41	9·7441114
X_4	37 41 00	−7	−8	37 40 45	9·7862113		+27	−2	−55	37 40 43	9·7862058
Y_4	54 35 09	−7	−7	54 34 55		9·9111284	+15	+2	+31	54 34 57	9·9111315
	360 00 55			180 00 00	9·3729367	9·3729009	175				
					9009						
					358						

$$\frac{358}{175} = 2''$$

Computation of sides.		Log lengths of sides.	
Log.			
sin X_1	9·8350688		
EF	3·4220740		
cosec Y_2	0·1286044	EC	3·3857472
sin X_2+Y_1	9·9999038	FC	3·5505822
sin X_2	9·8434437		
EC	3·3857472		
cosec Y_3	0·2558886	CD	3·4850795
sin X_3+Y_2	9·9902566	ED	3·6318924
sin X_3	9·9082004		
CD	3·4850795		
cosec Y_4	0·0888685	DF	3·4821484
sin X_4+Y_2	9·9766342	CF	3·5505822
sin X_4	9·7862058		
DF	3·4821484		
cosec Y_1	0·1537198	EF	3·4220740
sin X_1+Y_4	9·9960240	ED	3·6318922

$X_1 + Y_1 = 87°\ 44'\ 24''$
$X_3 + Y_3 = 87°\ 44'\ 16''$ } 8″

$X_2 + Y_2 = 92°\ 15'\ 25''$
$X_4 + Y_4 = 92°\ 15'\ 55''$ } 30″



$X_2 + Y_1 = 88°\ 47'\ 39''$
$X_3 + Y_2 = 102°\ 05'\ 29''$
$X_4 + Y_3 = 71°\ 22'\ 24''$
$X_1 + Y_4 = 97°\ 44'\ 29''$

To find the position of I.

TRIANGLE DFI.

	°	′	″	″	°	′	″
(1) DFI	85	31	24	+2	85	31	26
(2) FDI	37	11	57	+2	37	11	59
(3) DIF	57	16	32	+3	57	16	35
	179	59	53				

	°	′	″		°	′	″
Bearing FD	37	30	20	DF	217	30	20
DFI	85	31	26	FDI	37	11	59
Bearing FI	123	01	46	DI	180	18	21

log sin (1)	9·9986734	log sin bearing DI	7·7273601		
		log DI	3·5558771	log Diff. E	1·2832372
log DF	3·4821484	log cos bearing DI	9·9999938	log Diff. N	3·5587509
		log sin bearing FI	9·9234464	log Diff. E	3·2621158
log cosec (3)	0·0750553	log FI	3·3386684	log Diff. N.	3·0751206
		log cos bearing FI	9·7364522		
log sin (2)	9·7814647				

Co-ordinates D	460830·4		166793·7	F	458982·6		164386·1
Diff. E ...	−19·2	Diff. N	−3596·4	Diff. E	+1828·6	Diff. N	−1188·8
Co-ordinates I	460811·2		163197·3		460811·2		163197·3

Taking the mean of these values.

460811·2 163197·3

TRIANGLE EDI.

	°	′	″	″	°	′	″
(1) DEI	46	03	14	−2	46	03	12
(2) EDI	74	52	57	−3	74	52	54
(3) EID	59	03	57	−3	59	03	54
	180	00	08	−8	180	00	00

	°	′	″		°	′	″
Bearing ED=	75	11	02	DE	255	11	02
DEI=	46	03	12	EDI	74	52	54
Bearing EI	121	14	14	DI =	180	18	08

log sin (1)	9·8573241	log sin bearing DI	7·7222017		
		log DI	3·5558551*	log Diff. E.	1·2780568
log ED	3·6318923	log cos bearing DI	9·9999940	log Diff. N.	3·5558491
		log sin bearing EI	9·9319801	log Diff. E	3·6152135
log cosec (3)	0·0666387	log EI	3·6832334	log Diff. N	3·3980512
		log cos bearing EI	9·7148179		
log sin (2)	9·9847024				

Co-ordinates D	460830·4		166793·7	E	456688·4		165698·1
Diff. E ...	−18·97	Diff. N.	−3596·3	Diff. E.	+4123·0	Diff. N.	−2500·6
Co-ordinates I	460811·4		163197·4		460811·4		163197·5

Co-ordinates I from △ DFI...	460811·2	163197·3
Mean Value	460811·3	163197·3

* Compare value of common side DI from △ DFI.

ABSTRACT OF TRIG STATIONS.

Co-ordinates. East.	Co-ordinates. North.	Height.	Station.	E	F	C	D	A	B	H	G	I
456688·4	165698·1	147·5	E	×	3·4220740 119° 45′ 51″	3·3857472 30° 58′ 12″	3·6318923 75° 11′ 02″	3·6290720 16° 51′ 42″	3·7187307 42° 06′ 22″	2·9393419 77° 07′ 49″	3·3669729 67° 55′ 10″	3·6832334 121° 14′ 14″
458982·6	164386·1	110·4	F	3·4220740 299° 45′ 51″	×	3·5505822 342° 55′ 22″	3·4821484 37° 30′ 20″			3·3196198 316° 09′ 03″	3·3406574 356° 25′ 03″	3·3386684 123° 01′ 46″
457939·3	167782·4	125·8	C	3·3857472 210° 58′ 12″	3·5505822 162° 55′ 22″	×	3·4850795 108° 52′ 43″	3·2987331 359° 31′ 56″	3·4603270 51° 28′ 01″	3·2863199 192° 02′ 09″		
460830·4	166793·7	118·6	D	3·6318923 255° 11′ 02″	3·4821484 217° 30′ 20″	3·4850795 288° 52′ 43″	×	3·6192987 315° 41′ 22″	3·4559977 347° 11′ 46″			3·5558661 180° 18′ 15″
457923·1	169771·8	125·4	A	3·6290720 196° 51′ 42″		3·2987331 179° 31′ 56″	3·6192987 135° 41′ 22″	×	3·3583173 94° 48′ 50″	3·5909827 185° 41′ 40″		
460197·0	169580·3	132·4	B	3·7187307 222° 06′ 22″		3·4603270 231° 28′ 01″	3·4559977 167° 11′ 46″	3·3583173 274° 48′ 50″	×			
457536·2	165891·8	135·5	H	2·9393419 257° 07′ 49″	3·3196198 136° 09′03″	3·2863199 12° 02′ 09″		3·5909827 5° 41′ 40″		×	3·1691054 62° 30′ 31″	
458845·6	166573·2	117·7	G	3·3669729 247° 55′ 10″	3·3406574 176° 25′ 03″	3·1793858 323° 08′ 54″				3·1691054 242° 30′ 31″	×	
460811·4	163197·4	116·8	I	3·6832334 301° 14′ 14″	3·3386684 303° 01′ 46″	3·5558661 0° 18′ 15″						×

TRIANGLES FOR INTERSECTED POINTS.

Point.	<—— *Triangles.*		——>	*Common side.*	*Additional bases to be computed.*	*Computer's name and initials.*
1	*1 AC*	*1 BC*	*1 BE*	*1 C*	*EB*	*Sergt. Brown.*
				1 B		
2	*2 CA*	*2 BC*		*2 C*	—	
3	*3 CA*	*3 BC*		*3 C*	—	
4	*4 AB*	*4 BC*		*4 B*	—	
5	*5 CA*	*5 GC*		*5 C*	—	
6	*6 CA*	*6 AB*		*6 A*	—	
				7 A		
				7 C		
7	*7 CA*	*7 HC*	*7 HA*	*7 H*	*AH*	
8	*8 EC*	*8 HC*		*8 C*	—	*Bombr. Jones.*
9	*9 GC*	*9 CA*		*9 C*	—	
				10 D		
				10 I		
10	*10 FD*	*10 IF*	*10 DI*	*10 F*	—	
11	*11 GD*	*11 CD*		*11 D*	—	
12	*12 FG*	*12 HG*		*12 G*	—	
13	*13 IF*	*13 FG*		*13 F*	—	
				14 F		
				14 H		
14	*14 FG*	*14 HG*	*14 FH*	*14 G*	—	
15	*15 HF*	*15 GF*		*15 G*	—	*Gnr. Robinson.*
16	*16 EG*	*16 ED*		*16 E*	*EG*	
17	*17 HG*	*17 GF*		*17 G*	—	
18	*18 EH*	*18 EG*		*18 E*	*EH*	
19	*19 HE*	*19 GH*		*19 H*	—	
20	*20 CH*	*20 CG*		*20 C*	—	

COMPUTATIONS FOR POSITION OF INTERSECTED POINT No. 10.

Triangle IF 10. log base = *3·3386684*

Angle at F = From abstract :—

10 = *332 35 04*
I = *85 31 24*

247 03 40
Angle at F = *112 56 20*
Angle at I = *33 52 09*

sum = *146 48 29*

log sin 112 56 20 = *9·9642224*
log base IF = *3·3386684*
log cosec 146 48 29 = *0·2616591*
log sin 33 52 09 = *9·7460878*

3·5645499 = *I 10.*
3·3464153 = *F 10.*

Triangle DF 10. Base *3·4821484*.

Angle at D = *43 51 06*
„ F = *27 24 56*

sum = *71 16 02*
log sin 43 51 06 = *9·8406040*
log base = *3·4821484*
log cosec 71 16 02 = *0·0236378*
log sin 27 24 56 = *9·6631738*

3·3463902 = *F 10.*
3·1689600 = *D 10.*

COMPUTATIONS FOR POSITION OF INTERSECTED POINT No. 10—*contd.*

Triangle DI 10. Base *3·5558661.*

Angle at D = *81 03 03*
I = *23 24 23*

sum = *104 27 26*

log sin	81	03	03 = *9·9946808*		
log base			= *3·5558661*	*3·5645216* = *I 10.*	
log cosec	104	27	26 = *0·0139747*	*3·1689050* = *D 10.*	
log sin	23	24	23 = *9·5990642*		

Mean Values.

I 10 = *3·5645358*, D 10 = *3·1689325*, F 10 = *3·3464028.*

Computation of Co-ordinates from D 10 and F 10.

Bearing FD	= *37*	*30*	*20*	Bearing DF	= *217*	*30*	*20*
Angle DF 10	= *332*	*35*	*04*	Angle FD 10	= *43*	*51*	*06*
Bearing F 10	= *10*	*05*	*24*	Bearing D 10	= *261*	*21*	*26*

log sin 10 05 24 = *9·2435214* — *2·5899242*
log F 10 = *3·3464028*
— *3·3396334*
log cos 10 05 24 = *9·9932306*

log sin 261 21 26 = *9·9950401* — *3·1639726*
log D 10 = *3·1689325*
— *2·3458250*
log cos 261 21 26 = *9·1768825*

Diff. E = + *389·0*	Diff. N = + *2185·9*	Diff. E =	− *1458·7*	Diff. N = − *221·7*
Co-ord. of F = *458982·6*	*164386·1*	Co-ord. of D =	*460830·4*	= *166793·7*
459371·6	*166572·0*		*459371·7*	*166572·0*

Mean Value *459,371·6* *166,572·0.*

COMPUTATION OF POSITION OF POINT No. 16.

Solution of Triangles EG 16 and ED 16.

Triangle EG 16.

At E—					*At G—*				
Bearing of C	=	*30*	*58*	*12*	Bearing of F	=	*176*	*25*	*03*
Angle to 16	=	*104*	*04*	*18*	Angle to 16	=	*18*	*20*	*40*
Bearing of 16	=	*135*	*02*	*30*	Bearing of 16	=	*194*	*45*	*43*
Bearing of G	=	*67*	*55*	*10*	Bearing of E	=	*247*	*55*	*10*
Angle at E	=	*67*	*07*	*20*	Angle at G	=	*53*	*09*	*27*

log sin	*53*	*09*	*27*	= *9·9032457*	*3·3339190* = *E 16.*
log base				= *3·3669729*	
log cosec	*120*	*16*	*47*	= *0·0637004*	*3·3950915* = *G 16.*
log sin	*67*	*07*	*20*	= *9·9644182*	

Triangle ED 16.

At E—					*At D—*				
Reading of 16	=	*104*	*04*	*18*	Reading of E	=	*326*	*18*	*16*
Reading of D	=	*44*	*12*	*50*	Reading of 16	=	*296*	*04*	*12*
Angle at E	=	*59*	*51*	*28*	Angle at D	=	*30*	*14*	*04*

log sin	*59*	*51*	*28*	= *9·9369064*	*3·5687993* = *D 16.*
log base				= *3·6318923*	
log cosec	*90*	*05*	*32*	= *0·0000006*	*3·3339264* = *E 16.*
log sin	*30*	*14*	*04*	= *9·7020335*	

COMPUTATION OF POSITION OF POINT NO. 16—*contd.*

To find the height of I from D, E and F—

D	E	F
° ′ ″	° ′ ″	° ′ ″
ED +0 03 20	DE −0 19 47	EF +0 08 10
EI −0 04 20	EI −0 21 17	DI +0 07 55
2 ∣ 0 01 00	2 ∣ 0 41 04	2 ∣ 0 16 05
S = − 0 00 30	S = − 0 20 32	S = +0 08 22
L tan S = $\bar{4}$·1576244	L tan S = $\bar{3}$·7761907	L tan S = $\bar{3}$·3686227
Log DI = 3·5558661	log EI = 3·6832334	log FI = 3·3386684
log *h* = $\bar{1}$·7134905	log *h* = 1·4594241	log *h* = 0·7072911
h = − 0·517	*h* = − 28·8	*h* = + 5·1
Inst. & Beacon = + 0·325	Inst. Beacon = − 0·125	Inst. Beacon = + 3·2
− 0·192	− 28·93	+ 8·3
Height of D = 118·6	Height of E = 147·5	Height of F = 110·4
Height of I = 118·4	Height of I = 118·6	Height of I = 118·7
	Mean Value = 118·6	

To find the height of an intersected point (No. 10 from D and F).

D	F
° ′ ″	° ′ ″
Depression at D = −00 41 01	Depn. at F = −00 15 20
K ... ... = + 00 21	K ... = + 00 31
S ... = −00 40 40	S ... = −00 14 49
D.	F.
log tan S ... ... = $\bar{2}$·0729850	log tan S ... = $\bar{3}$·6344793
log D 10 ... ... = 3·1689325	log F 10 ... = 3·3464028
log *h* ... ... = 1·2419175	log *h* ... = 0·9808821
h = −17·5	*h* = − 9· 6
Instrument and Beacon ... ... = + 1·4	+ 1·45
−16·1	− 8· 2
Height of D = 118·6	Height of F = 110· 4
Height of 10 = 102·5	Height of 10 = 102· 2

Mean Value *102·35 metres.*

CHAPTER VII.

TRAVERSING.

25. Traverses with Theodolite and Measuring Tape.

1. A traverse is a connected series of straight lines (on the earth's surface), of which the lengths and bearings have been determined.

Traversing is ordinarily used for surveys in regions where the view is restricted, and where triangulation and ordinary plane-table work are consequently difficult or impossible.

For such work it is necessary, as in the case of triangulation, to grade the traverses in a descending order of accuracy.

In the most accurate forms the distances are measured with invar tapes, and the bearings deduced from measurements, with a suitable theodolite, of the angles at which adjoining lines meet. These bearings are checked at frequent intervals by astronomical observation.

In less accurate forms, distances may be measured with a steel tape or chain, by perambulator (cyclometer), or by subtense, or tacheometric methods (*see* Sect. 26).

The angles may be measured with a theodolite or compass, or drawn out graphically on a plane table.

For rough work distances may be paced or estimated by time.

2. In traverses for artillery purposes the angles should be measured with a small theodolite or director, and distances with a steel tape or chain, by subtense, or by tacheometric methods.

3. The errors to which theodolite traverses are liable are of two kinds, viz :—errors of direction, and errors of length.

The chief cause of errors of direction are :—

(*a*) Inaccurate centring of the instrument or signal over the station mark.

(*b*) Errors in reading the angles.

(*c*) Errors in bisection of the signal at the back and forward marks.

Errors in bearing or direction generated in each "leg" of a traverse, due to the above causes, are carried forward into those following it. The resultant error in direction consequently accumulates rapidly; great care must, therefore, be taken to keep the individual errors as small as possible by scrupulous care in centring the instrument or signal at each station, and by employing a sufficiently fine and distinct signal on which to align the theodolite.

(This is particularly important when the legs are short. For example, at a distance of 100 yards or less, the "signal" should be a nail or even a needle driven into a wooden picket.)

4. Linear errors in taping or chaining are due principally to :—

(*a*) Errors in length of the tape or chain used for measuring.

(*b*) Inaccurate placing of the end of the tape, *i.e.*, when the tape is carried forward from one position to the next and laid down again, the beginning of the tape in the second position may not exactly coincide with the position of its end in the first.

(c) Sag of the tape crossing uneven ground.
(d) " Gross " errors due to the inaccurate counting of the number of times the tape is laid down.

Various devices may be used for reducing these errors to a minimum. The following are the most usual and should be adopted whenever time and circumstances permit.

(i) The field tape should be frequently checked against a standard, and when in use kept at a constant tension.
(ii) The " markers," as described in Chap. V, Sect. **19**, should be used for marking the ends of the tapes.
(iii) Two measurements should be made of each leg using chains of different length for each, *e.g.*, one measurement might be made with a 100-foot tape and the other with a 66-foot tape or a 100-metre tape. The lengths of each leg as determined by the two tapes should be compared at the end of each measurement. If the discordance is great, the leg should be re-measured.

5. Traverses should always be run as closed circuits, starting from one previously fixed point and terminating at another, or at the starting point.

6. All stations should be carefully marked; a wooden peg with a nail driven into the head, or the socket of a " bearing picket " (Chap. XII, Sect. **48**) makes a good mark.

Special signals consisting of a rod or staff carrying a cross vane (black cross on a white ground or similar clear mark) should be provided to mark the back and forward stations. Observations should be made as low as possible on the staff, to the actual picket or nail if it is visible.

This practice will reduce errors arising from the staff not being vertical over the mark.

7. For traversing with a theodolite two parties are required: a theodolite party, and a taping party, the former consisting of an observer, a recorder and two signal men, and the latter of three tape men.

When the legs of the traverse are of considerable length, the theodolite party will probably work the faster, and should start first, the taping party following.

The parties should be separated by a clear interval of one leg, otherwise the tape men may get in the way of the signals, and the parties interfere with each other.

Stations should be carefully numbered so that there is no doubt about the correct co-ordination of the angular with the corresponding linear measurements.

The results should be recorded in a traverse book in the manner shown on page 46.

Vertical and horizontal angles should be observed, and observations invariably made on both faces of the theodolite.

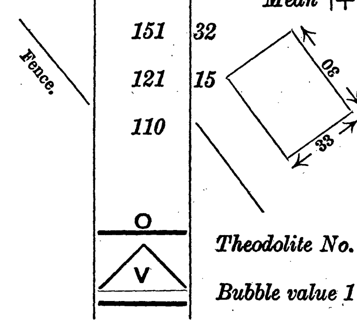
8. When the traverse has to be carried over very rough or swampy ground, distance should be measured by the methods described in Sect. **28**.

When only occasional obstacles are to be expected, steel tapes or chains should be used. Taping can be carried with fair accuracy across banks, escarpments, if not too high, by holding a pole vertically at the bottom or at one or two intermediate points on the bank and taping to the upper part of the pole holding the tape horizontal, and then lowering it to the bottom of the pole and continuing the measurement from there.

If the bank is too high or steep for this, it may be possible to measure to the edge and lay out a short base there from which the distance to a point beyond it or at the bottom can be triangulated.

Specimen of Traverse Field Book.

		(Horizontal.)			(Vertical.)			
		A and B. ° ′ ″	*Mean.* ° ′ ″		*C and D.* ° ′ ″	*Mean.* ° ′ ″	*O*	*E*
V	*F.L.*	*0 0 10* *0 20*	*0 0 15*		*4 30 0* *30 10*	*4 30 5*	*5*	*7*
VII	„	*122 12 10* *12 10*	*122 12 10*	VI				
			122 11 55	*320*				
V	*F.R.*	*255 1 20* *1 20*	*255 1 20*		*175 29 40* *29 50*	*175 29 45*	*8*	*4*
VII	„	*347 13 15* *13 5*	*347 13 10*			*4 30 15*	*13*	*11*
					Mean ...	*+4 30 10*		
			122 11 50		*Correction* ...	*+ 10*		
		Mean ...	*122 11 52·5*		*Corrected Mean*	*+4 30 20*		



Theodolite No.

Bubble value 1 Division = 20″

N.B.—For Artillery purposes it will not usually be necessary to fix the position of topographical detail crossed by the traverse line. When it is necessary to do so it should be entered on the traverse book as indicated above.

Rays observed to points which it is desired to fix by intersection from the traverse line should be included in the round of horizontal angles and entered up in the left-hand column.

26. Subtense Methods and Tacheometry.

1. In subtense methods the angle subtended by a bar of fixed length is measured with a theodolite. It is, however, equally possible to place cross wires in the eyepiece of a theodolite so that they define a fixed angle. The length subtended by this angle can be read off on a graduated bar and the distance of the bar from the observer deduced from these readings.

2. The methods using a fixed angle and graduated bar are known as *tacheometer* methods, in contradistinction to *subtense* methods, which imply a fixed bar and a measured angle.

Owing, however, to the optical properties of the ordinary telescope, the point at which the angle subtended by the cross wires is constant is not at the vertical axis of the telescope but at a distance $f + c$ in front of it, where :—

f is the focal length of the telescope (at solar focus), and
c is the distance of the centre of the object glass from the vertical axis.

Hence, if d is the required distance from the vertical axis of the theodolite to the centre of the bar held vertically while the telescope is horizontal, and if a is the reading on the bar,

$$d = k\,a + (f + c)$$

k being a constant depending on the fixed subtended angle.

It is necessary to know f and c, when k can most conveniently be determined by actual experiment, taking several readings of the bar at varying distances measured from a point $(f + c)$ in front of the vertical axis.

If the bar (held vertical) is on a different level to the theodolite so that the latter when pointed at the centre of the bar is inclined at an angle α to the horizontal, then :—

$$d = k.a. \cos \alpha + (f + c) \cos \alpha.$$

The difference in height is given by

$$h = k.a. \frac{\sin 2\alpha}{2} + (f + c) \sin \alpha.$$

3. The rapid and effective use of the tacheometer, or "stadia" as it is sometimes called, thus involves the employment of "tables," of which there are several in existence.

The method, which is not quite so straightforward as might appear at first sight, is used more in America and in the colonies than in England. In the hands of a man practised in its use it gives good and rapid results, the limiting length of the distances to be measured being from 600′ to 800′.

4. In American practice it is used to a considerable extent in conjunction with the plane table, not only for traversing but for ordinary survey. The procedure is as follows :—

Two men are required; one sets up and orients and resects the plane table position, using a telescopic alidade provided with stadia wires. The second man carries the graduated bar successively to surrounding points which have to be fixed.

These are fixed by a single ray and measured distance deduced from the readings on the bar.

5. The telescope of the alidade is provided with an additional converging lens which eliminates the quantity $(f + c)$. The same optical arrangement should be applied to any instrument designed for tacheometer work.

With such an instrument d varies directly with the reading of the bar, so that using the same notation as before—

$$d = k.a. \cos^2 a$$
$$\text{and } h = d \tan a.$$

It is, nevertheless, desirable in this case also to use tables instead of computing each distance.

The method is, however, only suitable for short distances, and it is doubtful if it will be of as much value to the artillery surveyor as subtense methods or others previously described.

6. Determination of distance by "subtense" methods is effected, as the name implies, by measurement of the angle subtended at the observer by a short measured length.

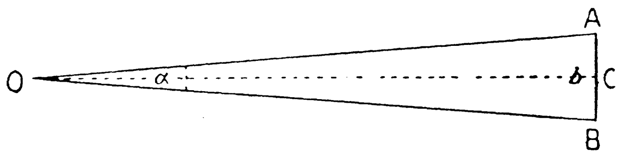


FIG. 23.

For example, in Fig. 23, if AB is a bar of known length held horizontally, and O the position of the observer. If OC is at right angles to AB then clearly

$$OC = \frac{AB}{2} \cot \frac{AOB}{2}$$

If the length AB is known and the angle AOB observed, the length of OC can easily be calculated.

7. Subtense methods have been used not only for measuring the lengths of traverses, but also for measurement of longer distances, as bases for a triangulation.

For traverse work the distance AB is usually represented by a bar with 2 fixed discs on it at a known distance apart, or by a graduated bar such as a levelling staff held horizontally or mounted horizontally on a tripod stand.

For longer distances the subtense base may be represented by two signals, one at each end of a 100′ or 300′ steel tape, or at any accurately measured distance apart. They should be aligned with a theodolite, or by taping, at right angles to the line represented by OC in the figure.

8. The subtended angle may be measured in various ways, some of which require a special theodolite. With an ordinary theodolite the

angle can be measured on the horizontal arc in the following manner:—

When the theodolite is set up and levelled and the bar placed in position (or the signals as the case may be), then, by keeping the lower plate clamped, the left hand disc or signal is intersected with a vertical wire, and the horizontal readings recorded. The upper plate is now clamped and traversed on to the right hand signal with the slow motion screw. The horizontal readings are again recorded.

Now, with the slow motion screw of the *lower plate*, bring the vertical wire once more on to the first object, then keeping both plates clamped traverse the telescope on to the right hand signal again with the slow motion screw of the *upper plate*. Repeat this process say 9 times, making 10 movements in all of the wire across the length of the bar. Read and record the horizontal reading.

Subtract the first reading on the left hand signal from the final reading of the right hand signal. The difference will be 10 times the subtended angle. Divide the result by 10 and compare with the difference between the first two readings.

The angle for computation will, of course, be the first of these two. The comparison with a single measure of the angle is necessary to ensure that the number of measures of the angle have been counted correctly.

9. This method of measuring an angle is often useful when it is desired to measure with a greater degree of precision than the refinement of graduation of the instrument permits when a single measure is taken. For example, if it is desired to measure an angle to 1′ or 2′ with a No. 4 director, graduated only to 30′; if successive single measures are taken it will not be possible to read to less than, say, 15′ and each reading will inevitably be to some extent biassed by those preceding it. By repeating the measures in the manner described above, and dividing by the number of repetitions, this bias is to a great extent eliminated.

When using a theodolite in this way the number of repetitions will depend on the length of the tangent screw, the accuracy required, and the time available.

Since the subtended angle is always small a very small error in its measurement has a large effect on the deduced length. The number of repetitions should, therefore, be as great, within reason, as time permits.

Using 10 repetitions on a 10′ bar with 8″ discs and a 5″ vernier theodolite, distances up to 1000 yards can be measured with an accuracy of about 3 yards. If greater accuracy is required, two or three sets of repetitions should be taken. For longer distances the length of the base should be such that the subtended angle is not less than 10′.

10. It is desirableth at the number of repetitions should be not less than five and this limits the practical length of the base even when the nature of the ground might permit a greater length than one subtending, say, 15′ or 20′ being used.

11. An alternative method of measuring the subtended angle is by means of an eyepiece micrometer moving a wire in the diaphragm. This is somewhat quicker than that described above, but requires a special theodolite.

The wire is made to intersect the discs on a bar in the same way as described above, and the angle is obtained from the difference in readings on the micrometer head, the repetitions being made as before.

It is necessary, however, to prepare a table beforehand, by actual experiment, giving the distances corresponding to a series of intervals of the micrometer. The reason for this is that owing to the optical principles of the telescope the angles measured in the eyepiece cannot be taken as those subtended at the vertical axis of the instrument, but at a point in front of this, when the telescope is set at solar focus.

It can be shown that if M is the micrometer reading, and d the required distance from the vertical axis of the telescope, that

$$d = \frac{A}{M} + B,$$

where A and B are constants whose values should be determined by actual experiment.

Example.—Referring to Fig. 23, suppose AB = 10 metres, and the first reading on A is 0° 4′ 20″, and after coming on with the crosswire to B once the reading on B is 0° 16′ 00″. After 10 measures the final reading is 1° 59′ 00″.

$$\text{Then } a = \frac{1^\circ\, 59'\, 00'' - 0^\circ\, 4'\, 20''}{10}$$

$$= \frac{1^\circ\, 54'\, 40''}{10}$$

$$= 11'\, 28''$$

$$OC = \frac{AB}{2} \cot {}^a/2$$

$$= 5 \cot 5'\, 44''$$

$$\log 5 = 0{\cdot}6989700$$

$$\text{Log} \cot 5'\, 44'' = 2{\cdot}7778663$$

$$\text{Log OC} = 3{\cdot}4768363$$

$$\therefore OC = 2998 \text{ metres.}$$

29. Computation of Traverses.

1. Traverses are computed from the formulæ—

$$x = x_1 + l \sin a,$$
$$y = y_1 + l \cos a$$

Where x_1 and y_1 are the east and north coordinates of the starting point or first station A, x and y those of the next station B, in front of x_1 y_1, l the measured distance between stations and a the bearing AB.

The bearings are deduced from an initial bearing or azimuth at the starting point and the observed angles. To obtain the corrected bearings for computation, it is necessary to "close" the traverse on the starting point and finishing point.

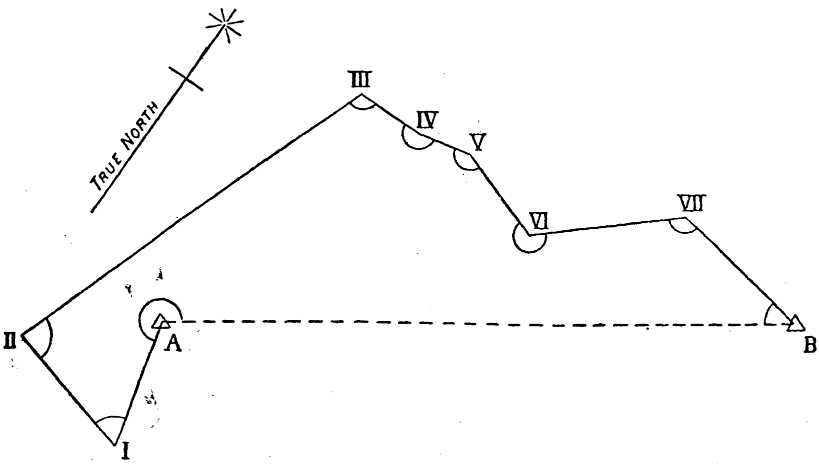


FIG. 24.

Thus in Fig. 24, if A and B are two trig points which are intervisible, the length and bearing of AB will be known, and the traverse legs, with AB, form an enclosed polyglon.

The sum of all the interior angles plus four right angles should equal twice as many right angles as the figure has sides.

This furnishes a check on the observed angles, which should be corrected to satisfy this condition.

2. The bearing of each leg should then be calculated by adding or subtracting the observed angle to the bearing of the preceding leg. Bearings should be reckoned clockwise from grid north, and the term $l \sin a$ or $l \cos a$ in the above equations given its appropriate sign.

3. The measured distances should be reduced to the horizontal by means of the formulæ.

Horizontal distance = measured distance × *cos* (elevation or depression).

An example of the computation is given on page 77.

4. Traverse bearings may be checked at intervals by astronomical observation of the azimuth of one leg. In topographical work, where traverse lines may be some miles long, the direction is usually checked astronomically every 15 or 20 stations.

5. When astronomical observations are used thus, the convergence of the meridians must be allowed for. An astronomical observation determines the direction of a line with reference to the true north at that point.

As explained in Chap. II, the true north lines on a map converge to a point at angles depending on their difference in longitude and on the particular projection in use.

6. The exact formulæ, by which the convergence of meridians is computed, will always be obtainable at the commencement of any operations from the R.E. Field Survey Services.

7. When the traverse has been computed as far as the co-ordinates of each station, the next step is to close it finally on the terminating point, and to correct the measured linear distances and the co-ordinates of each station to correspond with these.

The adjustment of traverse errors on a mathematical basis is very complicated, and is unnecessary for artillery purposes. Corrections should be deduced from the following empirical rules—

If n is the number of legs of the traverse,
y the total resultant error in northing,
x ,, ,, ,, easting.

The corrections to be applied to the co-ordinates of each station in order from the commencement will be

$$y/n,\ 2y/n,\ 3y/n\ \ \dots\ \dots\ \dots\ \frac{ny}{n}$$

and

$$x/n,\ 2x/n,\ 3x/n\ \ \dots\ \dots\ \dots\ \frac{nx}{n}$$

This is equivalent of course to distributing the total error uniformly throughout all stations. It is theoretically unsound, since it is an ordinary assumption, which is in most cases justified, that the error in the measured length of any ray varies as the square root of the length; it is nevertheless sufficiently accurate for practical purposes.

TRAVERSE.

Computation of Co-ordinates.

Starting Point .. A } mutually visible.
Closing Point .. B }

Data :—
Hor. distance AB = 1995·78 metres.
Bearing A to B = 59° 04′ 33″.

1	2	3	4	5	6	7	8	9	10	11	12	13	14		15		16		17	
Legs.	Included angle at the first station of each leg.	Angular error Correction.	Corrected Angle.	Bearing of Leg.	Vertical angle at first station of each leg.	Measured length in metres.	Horizontal distance in metres.	Co-ordinates of second station of each leg referred to the first.				—	Corrected co-ordinates from A.							
								Easting.		Northing.			Correction to Easting		Easting		Correction to Northing		Northing.	
								+	−	+	−									
	° ′ ″	″	° ′ ″	° ′ ″	° ′ ″															
A—B	0 00 00	+		59 04 33			1995·78	1712·08		1025·64			+	−	+	−	+	−	+	−
A—I	240 16 07·5	1 6·7	249 17 14·2	169 47 19	E 3 16 11	429·9	429·20	76·09			422·40	Closing errors E − 2·91 N + 1·29 corrections E algebraically " N " + −	0·364		76·45		0·172			422·57
I—II	60 24 00·0	1 6·7	60 25 06·7	289 22 12	E 0 10 20	476·0	476·00		449·05	157·87				0·728		372·23	0·345			264·87
II—III	86 22 52·5	1 6·7	86 23 59·2	22 58 13	E 0 41 40	1362·1	1362·00	531·53		1254·00			1·091		159·66			0·517	988·95	
III—IV	111 49 07·5	1 6·7	111 50 14·2	91 07 59	E 0 05 00	210·1	210·10	210·06			4·06		1·455		870·09			0·690	984·72	
IV—V	191 24 07·5	1 6·7	191 25 14·2	79 42 45	E 0 03 20	164·0	164·00	161·36		29·29			1·819		531·81			0·862	1013·84	
V—VI	144 06 45·0	1 6·7	144 07 51·7	115 34 53	D 2 01 33	320·0	319·80	288·45			138·09		2·183		820·62			1·035	875·58	
VI—VII	237 48 07·5	1 6·6	237 49 14·1	57 45 39	D 4 30 20	501·5	499·95	422·87		266·70			2·546		1243·86			1·207	1142·10	
VII—B	133 47 00·0	1 6·6	133 48 06·6	103 57 32	E 4 56 35	483·9	482·10	467·86			116·29		2·910		1712·08			1·380	1025·64	
B—A	44 51 52·5	1 6·6	44 52 59·1				+	2158·22	+	1707·86	680·84									
Total	1259 50 00						−	449·05	−	680·84										
Correct	1260 00 00						+	1709·17	+	1027·02										
	9)0 10 00							−2·91		+1·38										

0 01 06·7 to be added to six angles.
0 01 06·6 to be added to three angles.

Explanation of form of traverse computation.

Columns 2, 6 and 7 are filled in from the field books.

Column 4 is obtained by adding columns 2 and 3.

Column 5 is filled up by subtracting the figures in column 4 from those in column 5 in the next row above, adding 180° if the angle is less than 180°, and subtracting 180° if the angle is more than 180°.

e.g., 169° 47′ 19″
∠ − 60 25 07
109 22 12 + 180° = 289° 22′ 12″.

and 289 22 12
∠ − 86 23 59
202 58 12 − 180° = 22°. 58′ 13″.

Columns 8, 9–12 are calculated as below.

Calculation of column 8.

Horizontal distance = measured length × cosine vertical angles.

Examples—

Leg A — I log 429·9 = 2·6333674
log cos 3° 16′ 11″ = 9·9992924

Log Hor Dist. = 2·6326598
Hor Dist =429·20 metres.

Leg VI—VII
log 501·5 = 2·7002709
log cos 4° 30′ 20″ = 9·9986558

Log Hor Dist = 2·6989267
Hor Dist. =499·95 metres.

Calculation of Columns 9–12.

Difference of Easting = Distance × sine bearing.
Difference of Northing = Distance × cosine bearing.

Examples—

——	A–B	A–I	I–II
(1) log sin bearing	9·9334105	9·2486606	9·9746942
(2) log Hor Distance	3·3001127	2·6326598	2·6776070
(3) log cos bearing	9·7108813	9·9930659	9·5207025
log Easting (1) + (2)	3·2335232	1·8813202	2·6523012
log Northing (2) + (3) ...	3·0109940	2·6257257	2·1983095
Easting	1712·08	76·09	449·06
Northing	1025·64	422·40	157·87

The log Horizontal Distance can be taken from the working for column 8.

The sign of the easting and the northing can be seen from a diagram.

Column 14.

These are the corrections to be applied in each leg to the co-ordinates of the stations.

The corrections are all algebraically +, but since in the second leg the change in co-ordinate is —, *i.e.*, to obtain III from II the Easting 449·05 has to be substracted from the Easting of II, the correction of +·728 will diminish the figure 449·05 which must be *decreased numerically*. The correction must therefore be subtracted from the figure 449·05.

Columns 15 *and* 16.

Are obtained from columns 9–14, *e.g.*—

$$76{\cdot}45 = +\ 76{\cdot}09 + {\cdot}364 \text{ and is positive.}$$
$$372{\cdot}23 = 76{\cdot}09 - (449{\cdot}05 - {\cdot}728)$$
$$= 76{\cdot}09 - 448{\cdot}32 \text{ and is negative.}$$

CHAPTER VIII.

PLANE TABLING.

28. General Description and Preparatory Work.

(*See also* Manual of Map Reading and Field Sketching.)

1. The plane table is a drawing board, which can be mounted on a tripod stand, enabling it to be quickly set and roughly levelled, and also to be rotated about a vertical axis. It is used in conjunction with a sight rule or alidade, and a trough compass.

2. The process of plane-tabling is virtually triangulation done graphically *in situ,* instead of by observation and subsequent calculation. In suitable country and in skilled hands it is the quickest method of fixing points, and is at the same time susceptible of very considerable accuracy. It is the most suitable instrument, in ordinary open country, for fixing the position of our own guns.

3. Graphic work of any kind, however carefully done, cannot be expected to be as accurate as that done by theodolite observation. Unless adequate trigonometrical "control" is provided beforehand, errors will soon accumulate even in the most carefully executed plane-tabling; and such errors, even though they may appear small in themselves, are always exceedingly troublesome to the plane tabler.

When based on accurate and sufficient trigonometrical points, a point fixed by the plane table should not be in error by a greater amount than the breadth of a sharp pencil point, say 5 to 10 yards on a scale of 1/20,000.

4. The chief disadvantage of the plane table is that the paper mounted on it is speedily damaged by rain. Various substitutes have been tried, such as white zylonite sheets and grained zinc plates. None of them are very satisfactory. The lines are difficult to see, while the plates are apt to bend or buckle. If there is the least warp or

unevenness in the surface of the table, the sight-rule will rock about the raised parts and it will be most difficult to hold it steady or to rule lines accurately along its edge. The same applies to paper which has not been properly stretched. For this reason, except for purely temporary or very hasty work, the paper mounted on the plane table should be thoroughly stretched. The best hand-made drawing paper, "linen-backed" if possible, should always be used for plane-tabling, and should be mounted in the following manner :—

5. (1) Cut out a rectangle of paper about 6 inches larger in each dimension than the table top, and out of each corner cut a piece as shown in the figure by the shaded portions (Fig. 25.)

NOTE.—The shaded portions of linen and paper should be cut away.

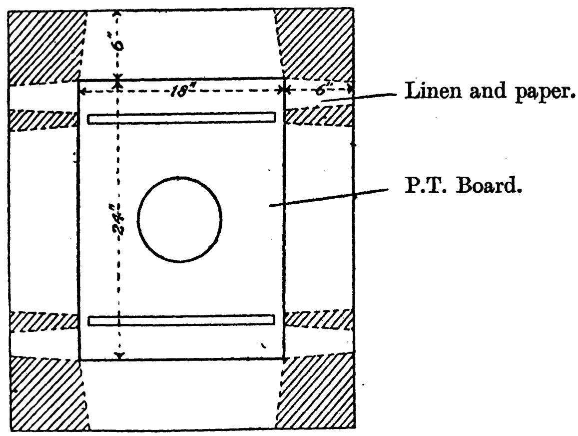


FIG. 25.

(2) Soak the paper in clean water long enough to soften it thoroughly without reducing it to a pulp or spoiling the surface.

(3) Place the paper face* downwards on a table covered with a clean sheet of cloth, or another sheet of clean paper, and lay the plane table top, also face downwards, centrally on it.

(4) Cover those portions of the paper projecting outside the plane table top with paste, and draw them over the edge of the board. Start with one side. Stick the paper to the under surface of the table top (the uppermost surface as it lies face downwards on the table), and stick four or five drawing pins in to hold it in position.

Then stick and pin down the other side, and afterwards the two ends, stretching the paper in the process.

(5) Allow the whole to dry.

6. When linen backed paper is not available the board should be mounted with linen or coarse muslin in the above manner and the paper, cut to the exact size of the board and not as in para. 5, mounted on it afterwards. In mounting the paper, the whole surface of the linen on the face of the board should be covered with paste, which should be well rubbed in (taking care that no lumps, however small, remain),

* *N.B.*—The correct face is found from the watermark. The drawing surface being that on which the watermark reads correctly.

and the paper pressed on to it with a damp cloth or sponge, taking care not to rub or damage the surface in doing so, and seeing that no "bubbles" are left under the paper. When the board is likely to be in use for some time, the upper edges should be protected with narrow strips of coarse paper stuck down along them. If no special paste is available, paste should be made from the finest flour—cornflour if possible—and clean water.

Paper mounted on linen gives a better surface to work on than paper mounted direct on the board. It is more durable and less likely to cockle on account of wet or damp.

29. Drawing the Grid and Plotting Points.

1. As soon as the paper on the plane table is quite dry, the plotting of the grid and trigonometrical points can be commenced.

It is not necessary, nor is it always desirable, to draw the grid parallel to the sides of the board. The first consideration in plotting the grid is to get as many trig points as possible on to the board consistent with having sufficient room available on the paper, not only for work immediately required, but for possible subsequent extensions from it.

It often happens that certain of the trig points are much more conspicuous than the others, and it is generally an advantage to project the grid on the board in such a way as to include these points, even when they are some distance away from the actual area in which the work is proceeding. As will be seen later, a distant and conspicuous point is very useful both from the point of view of accuracy and speed.

2. The grid should be drawn in with the aid of two beam compasses as follows :—

Having decided roughly the position of the grid from the considerations given above, select a point for the corner of one of the squares, say in the S.W. corner of the board, and rule in one of the grid lines through it.

Along this line scale off a distance equal to 4 grid squares or any multiple of this number, and mark each end of this length with a small needle hole. Set one beam compass to a distance equal to 3 (or the requisite multiple of 3) grid squares, and the other to 5 (or the proper multiple thereof), and with each compass draw two short arcs intersecting each other opposite each end of the marked length.

The intersection of these arcs (3, 4, 5 being the sides of a right-angled triangle) give the correct corners of other squares of the grid, which can then be drawn in. This method is much more accurate than attempting to lay off right angles with an ordinary protractor.

3. The grid should be drawn in in pencil first to make sure that all the necessary trig points come on to the board. It should be carefully checked to see that no mistakes have been made, and then ruled in in ink as finely as possible.

The co-ordinate numbers of the grid lines should then be entered in pencil close to the margins, and the square numbers written in, also in pencil, in the S.W. corner of each square. It is not, as a rule, necessary to ink these in unless it is certain that the board will be in use in the same area for some time.

4. Having entered up these figures, the trig points can be plotted. This should be done with a boxwood scale or by means of dividers and a specially drawn diagonal scale at the side of the board. This is preferable in every way to plotting by means of a co-ordinate card.

The points should be marked by drawing in about half an inch of the N. and S. and E. and W. lines passing through them, and pricking the paper exactly at the intersection of these lines with a fine needle.

Pins should never be stuck into the face of the board either for plotting or at any other time.

Plotting should always be very carefully checked before commencing outdoor work, and a list of all trig points plotted on the board, with their co-ordinates and heights, should either be written out on the board itself or entered in a notebook, and carried out to work at least until all points have been checked on the ground.

5. The height of each point should be written in beside its plotted position. If the point is some object, such as a windmill, of which the heights of both top and ground have been fixed, both heights should be entered thus :—

$$\frac{245\ \text{T}}{232\ \text{G}}.$$

Last of all the magnetic variation from grid north should be drawn in in a convenient place near the side of the board, which is then ready to take into the field.

6. Since, for artillery work in the field, the same board may be required successively in several positions at short intervals, all work except the grid should be left in pencil as long as possible, so that it can be rubbed out if necessary and the board used again in another position.

7. If it is desired to erase any marks entered in ink, this may be done by careful scratching with a very sharp penknife. This is generally preferable to "ink-eraser" rubber, which usually leaves a dirty mark on the paper. To erase with a knife the surface of the paper should be scraped off very gradually, using light pressure, until the ink marks have been cleaned off. The place should then be cleaned with a soft rubber and the surface of the paper polished up with the back of the finger-nail or the bone handle of a penknife.

If carefully done, good hand-made drawing paper will stand considerable erasure of this kind without becoming unserviceable.

30. Field Work.

1. The following instruments etc., are required for plane tabling.

(*a*) Plane table, stand and cover.

(*b*) Alidade, which should be at least as long as the longer side of the plane table.

(*c*) Trough compass.

(*d*) Clinometer.

(*e*) Boxwood scale (unless the alidade is scaled, when it may be dispensed with).
(*f*) Two hard and one soft pencils (HHH and HH for hard and HB for soft).
(*g*) Soft rubber.
(*h*) Dividers.
(*i*) Penknife.
(*j*) A small piece of emery paper or carborundum stone, for sharpening pencil points. (The emery paper on a small matchbox is very suitable.)
(*k*) A few spare drawing pins.
(*l*) A height indicator.

In tropical climates and for work in bright sunshine a pair of tinted glasses is also necessary, as the glare from the white paper is very trying to the eyes.

2. The object of the plane table is the fixation of points. When used for detail survey for mapping, the surveyor fixes sufficient points with the plane table to enable him to sketch in accurately, by eye, the whole of the country to be mapped, in exactly the same manner as an artist draws in a picture.

The more skilful the surveyor as an " artist," the fewer will be the points he requires to guide his hand and eye in copying the ground, and the quicker he will get over the ground. The good draughtsman, therefore, gains over the indifferent draughtsman in speed and finish, but not in accuracy. To be an accurate plane tabler there is no necessity to be a good freehand draughtsman.

3. The artillery surveyor will seldom have to use a plane table for mapping; but, since it is only by actually using it for this purpose that its possibilities and limitations can be properly realised, every artillery surveyor who has to make use of the plane table should in the first instance learn *how* to use it by making plane-table sketches of different kinds of country. The exact manner in which this should be done is largely a matter for the surveyor himself, who, with a little experience, will soon find the method of work by which he makes the best progress. The description given below shows the procedure usually adopted.

4.—(*a*) On first going out into the field, set up the plane table at one of the trig points, either in or close to the area of work. The point selected should be one from which there is a good all-round view, and the plane table should be set up as close to it as possible consistent with being able to walk round it without knocking the legs.

(*b*) Hold the pencil in the right hand and work the sight rule with the left. *Always work with the same edge of the sight rule.* This will generally be the right-hand edge looking towards the vane. See that this edge is not dented or damaged before coming out.

(*c*) The first consideration on putting up the plane table at any point is to orient the board correctly. This may be done in the first instance by means of the trough compass and the magnetic variation line ruled on the board.

This will, as a rule, only give an approximately correct orientation. Now lay the sight rule on the board between the point at which the P.T. is set up and the most distant visible trig point on the board. Test the ruler with the pencil point to see if it is the same distance from each of the points, and look through the sighting slit. If the point is visible it should be on or close to the wire of the sight vane. Loosen the clamping screw and rotate the board until the vane is laid on the point and clamp again, taking care that the sight rule is not moved in the process.

The board should now be correctly oriented or " set."

If a distant trig point can be identified without the aid of a compass the preliminary setting by means of the magnetic variation lines can be omitted.

(*d*) Having oriented the board, lay the sight rule successively between the P.T. position and every other trig point. The most convenient way of doing this, as of drawing any ray on the board, is to hold the pencil, point upwards, with its edge or circumference just touching the point from which the ray is to be drawn ; holding the sight rule in the left hand, keep the workiing edge pressed against the pencil, and, pivoting on this, lay on the other point. Test each ray with the pencil point to see that it passes *exactly* through the plotted positions of both points. The point of the pencil should be placed in the needle hole at *both* points and kept at the same slope for each pair.

If any point does not pass this test, first check the plotting of it, and if this is correct, mark it with a query and do not use it again. *There should be no errors in the positions of trig points, which can be detected with a plane table.*

(*e*) As soon as the points have all been checked and it is certain that the board is correctly oriented, the trough compass should be placed in a convenient position at the side of the board and turned round until the needle comes to rest at the centre of its run. A line should then be ruled round the edges of the box. It is not necessary to remove the compass from its box ; it is, in fact better, not to do so, as the removal from the box may injure the point of the pivot. It is sufficient to wedge the compass firmly into its box and mark the edges of the box. (The needle does not rest on the pivot until the lid is withdrawn ; this should not be done until the box is placed on the plane table.) The compass is now put away and the plane table is ready to commence the fixation of points.

(*f*) These may be fixed either by *intersection* or by *resection.* The process of plane tabling is really nothing more than the intelligent combination of both. The plane tabler sets up his board successively at different places, whose positions he determines by resection, and from each he draws rays to surrounding objects, whose positions are then fixed by other intersecting rays from other resected positions, or " fixings," as they are generally termed.

(*g*) As a general rule, it may be said that, when many points have to be fixed, the quickest work is done by the man who makes most use of intersection.

In surveying, rays should be drawn from each fixing to as many points as the surveyor thinks he can subsequently identify from other fixings. Along each ray so drawn a ring or mark should be made at the estimated position of the object. A description of the object may be written along the ray, but it is preferable to make use of simple symbols suggestive of the object. These take up less room and keep the table cleaner; after a little practice they convey actually more information than writing, which soon becomes confusing when many rays have to be drawn.

(*h*) The first care in plane tabling, particularly in close country, should be to fix by intersection as many prominent objects as possible, whether these are ultimately required for the map or not.

Each point should be fixed by at least three rays meeting exactly in a point. Points fixed by two "reliable" rays meeting at angles between 45° and 90° may be accepted "provisionally," but should be checked by a third ray as soon as possible.

Open ground, tops of ridges, etc., from which the view is good should be gone over first. In such places it will be comparatively easy to resect, as many trig points should be visible. From fixings taken in the open ground numerous points must be fixed by intersection in the more enclosed portions. It is waste of time trying to survey enclosed country by resection, unless numerous points can be and have been fixed in it If this is not or cannot be done, the surveyor will speedily find himself compelled to resort to plane-table traversing, which is much slower and less accurate than resection.

(*i*) Resection is the process of fixing a point by rays drawn from it to two or more previously fixed points (as opposed to intersection whereby a point is fixed by rays drawn *to* it and *from* two or more fixed points).

To resect a position the board must first be correctly oriented. This may be done in any of the following ways:—

(i) By means of the trough compass.
(ii) By "back" or "forward" ray.
(iii) By selecting a "trial" point and laying on a distant point.

Of these (ii) can only be relied on to give an accurate orientation at the first attempt. It is only possible to use it, however, when the site of the fixing can be selected at some previous station. When this can be done a ray is drawn to it and a short length of the ray ruled in also at the extremities of the sight rule. These are called "repere" marks; their object is to give a long line along which to lay the ruler in the setting the board at the next point. On reaching this point, the board can be set by laying the rule in a reverse direction along this ray, and laying on the first point. This is termed setting by "back" ray.

It cannot, of course, be used unless the first point is marked in some way. If there is no mark on it on which to lay, it may be possible to select some point exactly *beyond* the second point and use this in a similar manner, by setting "forward" instead of "back" ray. This is the quickest way of setting the board, and if done carefully is very accurate. Having thus set the board, its position can be fixed by a single ray drawn to, or back from, some point to a flank, intersecting the first ray at an angle of 45° or more. A ray from a third point should be drawn as a check. If these rays do not come to a point, the setting is incorrect and must be corrected by ordinary resection from three points as described below. This method of fixing a point is known as " countersection " and will be found especially useful on commencing work on a new board.

(*j*) Methods (i) and (iii) can only be relied on to give an approximate setting. The result of this is that when rays are drawn back from three fixed points they do not come to a point, but give a triangle of error, from which the true position of the " fixing " must be estimated and the setting corrected until all rays come to a point.

The following rules enable the true position to be estimated :—

(i) The true point must be placed so that it is either to the right or to the left of all the rays when facing the fixed points. In other words, imagining each ray pivoted at the position of the point, they must be brought together by moving *all* clockwise or *all* counter-clockwise.

(ii) The perpendicular distance of the true position from each ray is proportional to its distance from the corresponding fixed point.

From the first rule it follows that, if the position of the fixing is *within* the triangle formed by the fixed points, the true position will be within the triangle of error. If it is outside the triangle formed by the fixed points, it is *outside* the triangle of error.

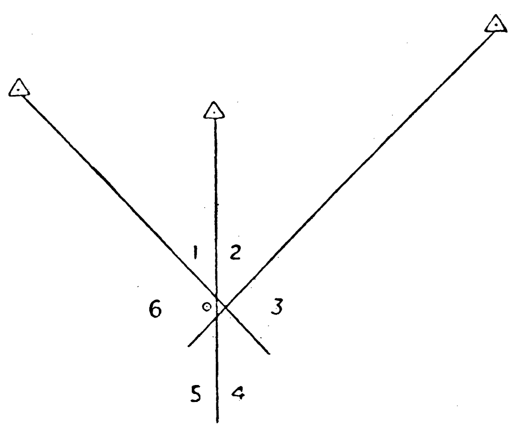


FIG. 26.

From Fig. 26 it can be seen that condition (i) in this case is only fulfilled by sector 6 and 3, and condition (ii) in sector 6.

Having estimated the true position from these two rules, the sight rule is laid along the line joining this to the most distant fixed point. The clamping screw is released, and the board swung round until the sight rule is laid on the point. The board is then clamped and all rays drawn in again, the process being repeated until all come to a point.

In order to resect without unnecessary loss of time the following points should be borne in mind.

(*a*) Accuracy of *position* is ensured by rays from near points. Accuracy of *setting* by aligning on distant points.
(*b*) Try to select points for resection so that two near points give rays cutting at 90°, and use the position so found in conjunction with a distant point to set the board.
(*c*) Select for resections, positions and points arranged so that a small error in setting gives a large triangle of error.

Figs. 27-28 show the triangles of error given by an "error" in setting of 5° for different positions of the fixing in relation to the fixed points, and the true position which would be deduced from a correct setting.

It will be observed that at X, as at any point on the circle ABCX, all the rays meet exactly in a point in spite of the error in orientation. In other words, when the fixing is on this circle called usually the "Danger Circle," the method fails. At any point near it, as at Y or Z, is is unreliable, even if great care is taken.

The arrangement of points in Fig. 27 should therefore be avoided if possible. It can be seen that with this arrangement a large error of setting gives quite a small triangle of error in almost all positions. It may therefore be thought that, when rays meet practically in a point the setting is correct, whereas neither the correct position nor the correct orientation has been obtained. Any ray drawn from such a fixation will be incorrect both as to position and direction.

It is a safe rule for all inexperienced plane-tablers to avoid "outside" fixings altogether. They should rarely be necessary if care is taken, as explained in para. 4 (*h*), to fix a sufficiency of plane table points by intersection in the earlier stages of the work.

Such plane table points should be marked with a dot surrounded by a small circle.

5. The above description refers particularly to plane tabling executed for mapping. The work of the surveyor fixing battery positions is, however, not greatly different. It differs in fact only in degree, and not in kind. Batteries are commonly sited in low ground, positions in the open are generally avoided. It is rarely possible therefore to fix the guns by simple resection at the battery position from the surrounding trig points.

In the majority of cases more or less preliminary plane table work, of the character described above, will be necessary before it is possible to resect or otherwise fix the gun positions. If this is to be done expeditiously, the surveyor must know how to set about this preliminary work, and to carry it out quickly and accurately.

6. In using the plane table the following general maxims should always be observed :—

(1) *When the plane table is set up and before work is begun.* See that the screws at the top of the legs are taut, and the legs firmly in the ground. Test by tightening the clamping

To face page 86.

Fig. 27.

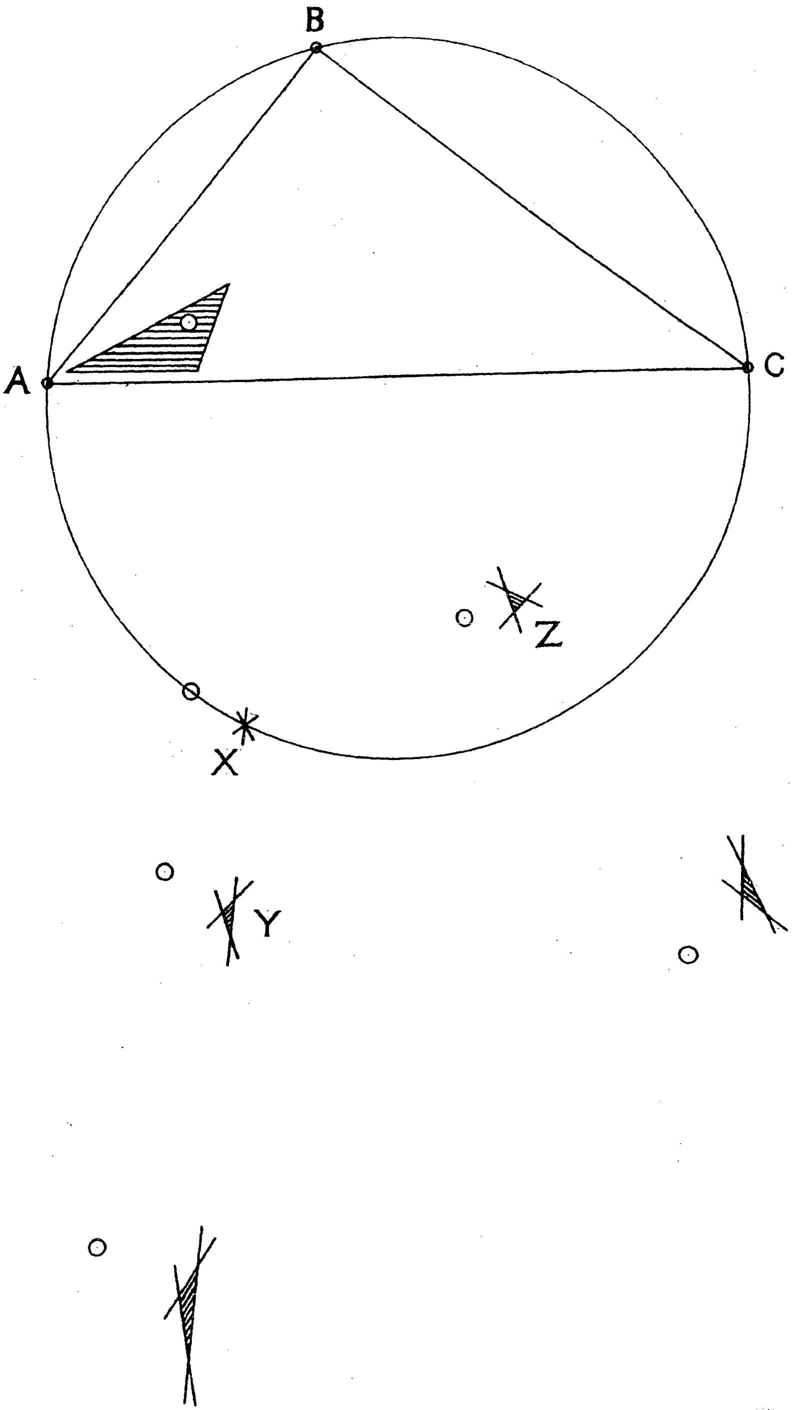


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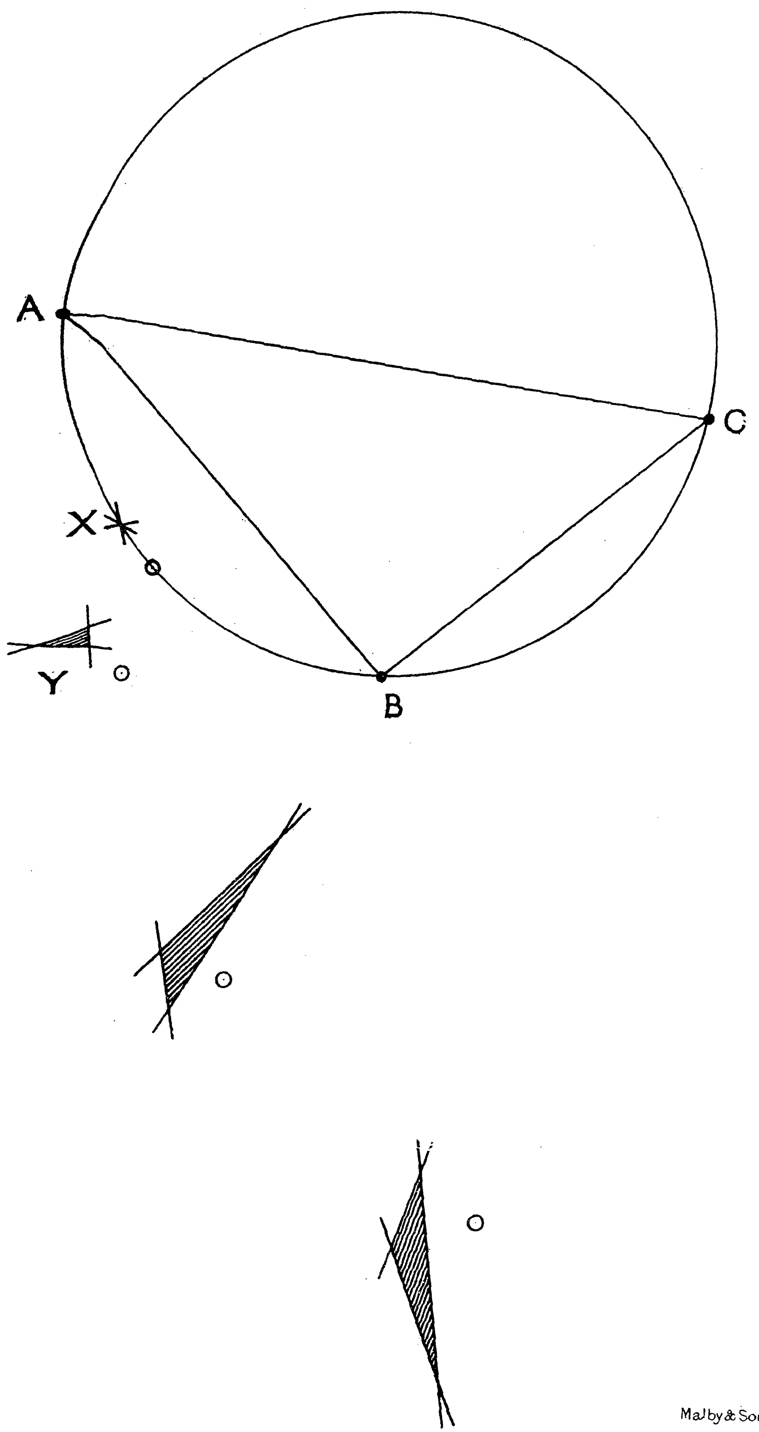
Fig. 28.



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screw and trying to rotate the board. If there is any shake or movement, look to the screws at the top of the legs. See that the board is as level as possible, and that the legs are not too close together.

(2) *During work.*—(*a*) Keep the pencil very sharp. Accurate work cannot be done with a blunt pencil.

(*b*) To draw a ray to or from a point, hold the pencil nearly vertical and bearing against the ruler, note the angle at which it must be held for the ray to traverse the point, and *keep it at this angle* while ruling in the ray. Without this precaution errors will soon creep into the work.

(*c*) Press lightly at all times.

Experience alone will show how easy it is, if care is not taken, to make small mistakes in so apparently simple an operation as resecting, and how soon these small mistakes accumulate into embarrassing errors.

The great advantage of combined resection and intersection is that the presence of such errors is quickly revealed and they can then generally be corrected. When, however, the country is too enclosed to permit of resection, recourse must be had to plane table traversing.

7. In this case the distance between successive fixings must be measured along the ground (preferably with a steel tape or chain as in theodolite traversing), and the board set at each fixing by back or forward ray. Each traverse should commence and terminate at an intersected or resected point or a trig point, and the closing error should be distributed throughout its length.

When time is of importance the board may be set by compass only. This is generally much less accurate than by setting by back or forward ray. For rough work, distances may be paced, but a bicycle fitted with a cyclometer, if available, is both quicker and more accurate; it should, however, be calibrated beforehand along accurately measured distances.

CHAPTER IX.

HEIGHTS AND CONTOURS.

31. Levelling.

1. The determination of the relative heights of two or more points may be done in one or other of the following ways:—

(*a*) By "levelling" with either a theodolite or "level."
(*b*) By observation of reciprocal angles with a theodolite.
(*c*) By means of a clinometer used in conjunction with a plane table.
(*d*) By aneroid or barometer.
(*e*) By hypsometer (or boiling point thermometer).

2. Of these the first three may be regarded as normal survey methods; the last two are relatively inaccurate, and are only used when the first three methods are inapplicable, as, for example, in exploration or reconnaisauce work.

3. The instruments required for "levelling" consist of a special levelling instrument or "level," consisting essentially of a telescope, mounted so that it can be revolved in azimuth, and provided with a specially sensitive bubble so that the line of collimation can be quickly and accurately levelled in any position, and one or two graduated staves.

These staves are usually from 10 to 15 feet long, and may be graduated either in feet and inches or in metres and decimals of a metre.

4. To determine the difference in level between two points, the instrument, mounted on a tripod, is set up and levelled at some convenient place, and pointed at the staff held successively in a vertical position over each point. The reading of the horizontal cross wire on the staff at each is noted, and the difference between these two readings gives the difference in height between the two points.

5. It is to be noted that—

(*a*) The height observations are independent of the height of the instrument.

(*b*) The instrument need not be set up over either point, hence no time is spent in "centreing."

(*c*) The distance of the instrument from the points is immaterial.

(*d*) Provided the distance of the instrument from both points is the same, the effect of refraction and curvature on each observation will be identical. The difference between the uncorrected observations will give the true difference in height.

6. When it is necessary to determine by levelling the difference in height between two widely separated points, it is necessary to select a number of intermediate points and determine the difference in height between adjoining points by successive and independent observations. The difference in height between the two terminal points will obviously be the algebraic sum of the differences in height between the intermediate points.

7. For ordinary survey work levelling is used to supplement and correct the heights determined by triangulation. Owing to the elimination of corrections for curvature and refraction, heights can be determined more accurately by levelling than by triangulation.

Levelling for this purpose is, like triangulation, graded in accuracy. Goedetic and precise levelling, executed with every possible refinement, corresponds to the primary triangulation and fixes the heights above a common datum of a number of stations scattered over the country. These points in their turn are connected by secondary or tertiary lines of level executed with less accuracy, and corresponding in their sphere exactly with the secondary and tertiary triangulation interpolated between and based on primary triangulation points.

8. For artillery purposes levelling will rarely or never be necessary, as the accuracy of even tertiary levelling is much greater than anything

ever likely to be required. Sufficient accuracy will always be attainable by observation of reciprocal angles with a theodolite computed in the manner described in Chapter VI, Section 25.

32. The Indian Clinometer.

1. Heights can be fixed sufficiently accurately for artillery purposes by means of the Indian clinometer and the plane table, provided the point to be fixed is not too far from some trigonometrically fixed point.

2. This instrument is normally used with a plane table for contouring, and consists of a brass bar 9 inches long attached at one end by a hinge to a flat brass plate of the same length and about one inch broad.

At the other end of the bar is a mill-headed levelling screw by means of which that end can be raised or lowered relatively to the hinge so that the bar can be brought to a horizontal position.

Attached by hinges to the upper surface of the bar are two metal vanes, 8 inches apart. The one fixed to the bar at the front end, above the hinge attaching it to the base plate, is about 7½ inches long and 1 inch wide and is slotted down the centre. On one side of the slot is engraved a scale of natural tangents, and on the other a scale of degrees.

The zeros of these scales are in the centre of the vane.

The other vane is placed just in front of the levelling screw and is provided with a sight hole which can be brought to the same level as the zero on the front vane by means of the levelling screw and a small level let into the centre of the brass bar.

3. To use the clinometer the two vanes are turned upright and the clinometer aligned on the object so that on looking through the sight hole the object appears in the central slot of the front vane. The level is then brought to the centre of its run by means of the screw, and the eye placed to the sight hole.

The natural tangent of the angle of elevation or depression to the object can be read off on the scale by estimation to three places of decimals.

The difference in height between the object and the observer can then be found by multiplying this natural tangent by the horizontal distance as scaled off from the plane table.

If the height of the object is known (*e.g.*, if the object is a trig point) the height of the plane table can be deduced, and conversely, if the height of the plane table is known the height of the object can be determined.

33. Contouring on the Plane Table.

1. Contouring with the plane table is essentially eye sketching controlled by numerous heights fixed with the clinometer. As a preliminary, it is necessary to fix, by intersection or otherwise, the positions of a number of points scattered over the area. These points may be bushes, trees, corners of hedges, huts, haystacks, &c., and should be selected as far as possible from those which are found at points where the slope of the ground changes.

2. If the height of two points, one at the top and one at the bottom of a uniform slope can be fixed, it is clear that the positions of contours can be easily and quickly interpolated with considerable accuracy along the line joining them. If a sufficient number of such points can be fixed over the whole area and heights interpolated in this way between them at suitable intervals, the points so found can be joined up by eye so as to give a contour accurately following the form of the ground.

3. In contouring by this method the following points should be noted :—

(*a*) The clinometer should invariably be carefully tested before going into the field. This may be done in several ways, described in paras. 4 and *et seq.*

(*b*) Heights should always when possible be fixed from at least two points, which should be so chosen that one of them is above and the other below the point to be fixed. (When the survey is based on accurate triangulation the mean of the two values so obtained should agree within 20 per cent. of the vertical interval between the contours.) The points chosen should be reasonably close to the point which has to be fixed. The Indian clinometer is not intended for use over long distances. Four or five inches on the paper may be taken as a normal maximum distance. (The actual distance on the ground will of course depend on the scale of the map.)

(*c*) The most difficult places to contour are the bottoms of valleys. The view from such places is always restricted, and it may be difficult to find positions where the plane table can be set up and fixed in them. It is better to try and fix points along the bottoms of the valleys by intersection from fixings on the surrounding spurs and deduce their heights from these.

(*d*) It is often easier to contour the face of a hill or slope from the hill or spur on the opposite side of the valley, than from the slope itself.

4. Testing of Indian clinometers may be effected in three ways :—

(*a*) By comparison with a theodolite.

(*b*) By reciprocal angles.

(*c*) By vertical angles observed with the clinometer from one trig point to another.

It is not as a rule necessary or desirable to alter the adjustment of a clinometer, which involves alteration of the setting of the level.

The testing of the clinometer should be directed to determining a correction to be applied to all observations taken with it.

5. *Comparison with a theodolite.*—Set up and level a theodolite and observe, on both faces, the vertical angles to two or three well-defined points at distances varying from 200 to 2,000 yards.

Look up the natural tangents of the angles so found in a book of tables.

Set up a plane table beside the theodolite at a height sufficient to bring the eye-hole of a clinometer placed on it approximately on the same level as the trunnions of the theodolite.

Take the readings of the same points with the clinometer on the natural tangent scale.

The difference between the observed values of the natural tangents and those taken from the table gives the correction to be applied to the former. The values of the correction deduced from the various points observed should not differ *inter se* by more than ·001.

6. *Testing by reciprocal angles.*—Set up two plane tables, preferably on sloping ground, about 100 to 200 yards apart, place a clinometer on each, and mark the position of the base plates of each.

Place the "traveller" of each clinometer scale in turn at the zero point, and with the other clinometer note the reading to it on the tangent scale.

Interchange the clinometers and repeat the observation.

With each clinometer, if the adjustment is correct, the angle of elevation observed from one end should be equal to the angle of depression from the other.

If this is not the case, the correction necessary is half the difference between the two readings.

Example:—

First observation gives elevation	·046
Second observation gives depression	·050
Difference	·004
Correction	·002

The mean reading is ·048, and the clinometer should have read this amount in both directions.

All elevations observed with this instrument are therefore ·002 too small, and all depressions ·002 too large (numerically).

If elevations are conventionally regarded as + and depressions —, the correction is + ·002, and must be added algebraically to all observations.

7. *Testing by observation from one trig point to another.*—The difference in height of two trig points will be known, and their horizontal distance apart can be scaled off from the plane table.

The first, divided by the second, gives the natural tangent of the angle of sight between them.

The tangent so found can be compared with that observed on the clinometer, and the correction found by subtracting one from the other.

CHAPTER X.

TRIGONOMETRICAL RESECTION.

34. General Principles.

1. Trigonometrical resection consists in the determination of the position of a point by calculations from the angles observed from it to surrounding trig points.

2. Trigonometrical resection is not very largely used in topographical surveys, as the bulk of the observations for such purposes are made from the trigonometrical stations themselves.

The artillery surveyor, however, is frequently called upon to establish trig points in positions convenient for the use of batteries (*see* Chapter XII) rather than for topographical work, and it is quicker and easier to establish the position of such a point, as soon as it has been selected, by observation made at it, than to "flag" the point and observe to it from two or more trig points.

3. The principles on which trigonometrical resection is based are identical with those on which plane-table resection depends, and the same rules as to selection of trig points holds good. The refinement of angular observation with a theodolite or director are, however, so much greater than anything attainable by graphic work that, practically, a satisfactory determination of position can be obtained anywhere except on or very close to the "danger" circle.

4. The "field work" in trig resection consists simply in the observation of a round of angles at the point to be fixed, to three or more surrounding trig points.

As a rule, in order to get a check on the result, four points should always be included in the round.

If the existing triangulation is of rather poor quality the round should include every visible trig point.

5. From this round of angles (which should normally include vertical angles so that the height as well as the position of the point can be deduced) the position of the point can be calculated by a number of methods.

6. These may be divided into two classes—

(*a*) *Absolute methods* in which one value of the position is deduced directly from the observations to *three* trig points.

(*b*) "*Trial point*" methods in which an approximate position is first determined graphically, or simply estimated, and the distance and direction of the true position from this approximate one is calculated.

7. The disadvantage of absolute methods lies in the fact that only three points can be used at a time and each three together may give a different value from each other three.

When a number of points have been observed, the computation has to be repeated several times, and the whole operation becomes very laborious.

8. With most of the trial point methods the computation is such that extra points can be brought into the work with little additional labour.

9. Examples are given below of two absolute methods and three trial point methods, two of which are semi-graphic, and one entirely logarithmic.

Each has its advantages and disadvantages, which will be compared when the methods have been explained.

Absolute Trigonometrical Resection.

Auxiliary Point Method.

	Trig. Point.	Observed angles.			Co-ordinates.		
					East.	North.	
A	*Beacon Tumulus TP*	*00°*	*00′*	*00″*	*461884·4*	*162687·0*	
B	*Pauper's Prospect TP*	*39*	*34*	*06*	*460158·6*	*161169·3*	α = 39 34 06
C	*King's Barrow TP*	*90*	*07*	*26*	*457409·8*	*161713·4*	β = 50 33 20
D	*Knighton TP*	*179*	*24*	*40*	*456688·4*	*165698·1*	

	E.	N.
Co-ords. A	*461884·4*	*162687·0*
„ C	*457409·8*	*161713·4*
	4474·6	*973·6*

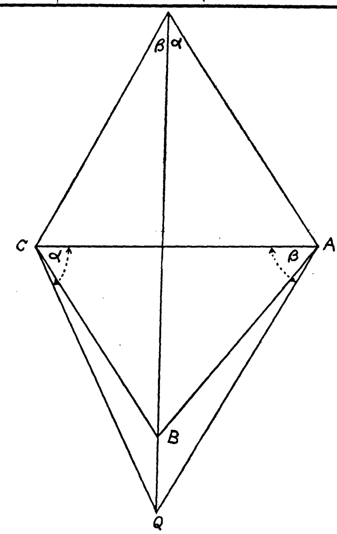
Log. DE = *3·6507542*
Log. DN ... *2·9883806*

Log. Tan θ = *10·6623736*

θ = *257° 43′ 29″*

Log. DE ...	*3·6507542*	Log. DN ...	*2·9883806*
Log. Sin. θ ...	*9·9899556*	Log. Cos θ ...	*9·3275812*
	3·6607986		*3·6607994*

Mean Log Distance AC = *3·6607990*



To find angle A.Q.C. = 180° − (α + β)

		180° 00 00
− (α + β)	=	*−90 07 26*
Angle A.Q.C.	=	*89° 52′ 34″*

To find Distance AQ and CQ.

Log. Sin α = *9·8041381*
„ Dis AC = *3·6607990* } = *3·4649381* = *AQ*
„ Cosec AQ̂C = *0·0000010*
„ Sin β = *9·8877528* } = *3·5485528* = *CQ*

To find Bearing AQ.

Bearing AC	*257°*	*43′*	*29″*
− β	*50*	*33*	*20*
AQ	*207*	*10*	*09*

To find Bearing CQ.

Bearing CA	*77°*	*43′*	*29″*
„ + α	*39*	*34*	*06*
„ CQ	*117*	*17*	*35*

	Bearing.	Computation.	Log Distance.	E.	N.	Co-ordinates. East.	Co-ordinates. North.
A to		Sin. *9·6595542* Dis. *3·4649381*	*3·1244923*	*1332·0*		A = *461884·4* *−1332·0*	*162687·0* *−2595·2*
Q	*207 10 09*	Cos. *9·9492252*	*3·4141633*	—	*2595·2* —	Q = *460552·4*	*160091·8*
C to		Sin. *9·9487419* Dis. *3·5485528*	*3·4972947*	*3142·6*		C = *457409·8* *+3142·6*	*161713·4* *−1621·6*
Q.	*117 17 35*	Cos. *9·6613790*	*3·2099318*	+	*1621·6* —	Q *460552·4*	*160091·8*

To find Bearing QB or QP.

	E.	N.
Co-ords. Q	*460552·4*	*160091·8*
„ B	*460158·6*	*161169·3*
	393·8	*1077·5*

Log. DE *2·5952757*
„ DN *3·0324173*

Log. Tan θ = *9·5628584*

θ = *339° 55′ 26″*

To find Angle QAP.

Bearing QA	...	*27° 10′ 09″*
„ QB	...	*339 55 26*
Angle AQB	...	*47 14 43*
+ α	...	*39 34 06*
		86 48 49
		180° 00′ 00″
Angle QAP	...	*93° 11′ 11″*

To find Angle QCP.

Bearing QC	...	*297° 17′ 35″*
„ QB	...	*339 55 26*
Angle BQC	...	*42 37 51*
+ β	...	*50 33 20*
		93 11 11
		180° 00′ 00″
Angle QCP	=	*86 48 49*

To find Distance QP.

Log. Sin QAP = *9·9993281*
„ Dis AQ = *3·4649381*
„ Cosec α = *0·1958619*

3·6601281

Log. Sin QCP *9·9993281*
„ Dis CQ *3·5485528*
„ Cosec β *0·1122472*

= Dis QP = *3·6601281*

	Bearing.	Computation.	Log Distance.	E.	N.	Co-ordinates of P. East.	Co-ordinates of P. North.
Q to		Sin. *9·5356335* Dis. *3·6601281*	*3·1957616*	*1569·5*	+	Q. *460552·4* *−1569·5*	*160091·8* *+4294·4*
P.	*339° 55′ 26″*	Cos. *9·9727754*	*3·6329035*	—	*4294·4*	P *458982·9*	*164386·2*

Absolute Trigonometrical Resection.

Method 1, Figs. 29 and 30 (Auxiliary point method).

A, B and C are three trig points, P the point to be resected, and α and β the angles observed at P.

Two cases have to be considered:—Case I where P lies within triangle ABC as in Fig. 29, Case II where P lies outside this triangle as in Fig. 30.

The *Construction* is as follows:—

Case I (Fig. 29). Draw CQ to meet BP in Q making $A\hat{C}Q$ equal to $180 - \alpha$. Then $A\hat{C}Q = 180 - \alpha = 180 - A\hat{P}B = A\hat{P}Q$.

$\therefore$ A, P, C and Q lie on a circle. Hence $C\hat{A}Q = C\hat{P}Q = 180 - \beta$.

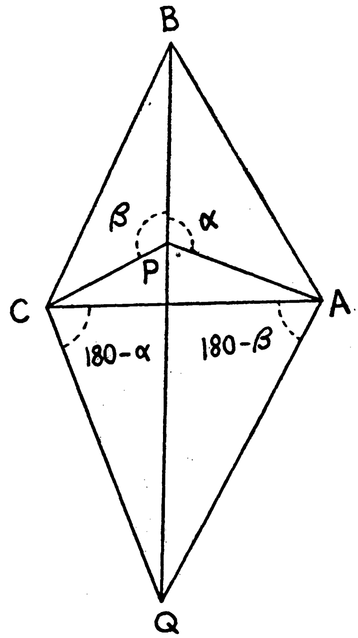


FIG. 29

Case II (Fig. 30). Draw CQ to meet PB in Q making ACQ equal to α. Then $A\hat{C}Q = a = A\hat{P}Q$. Therefore A, P, C and Q lie on a circle. Hence $C\hat{A}Q = C\hat{P}Q = \beta$.

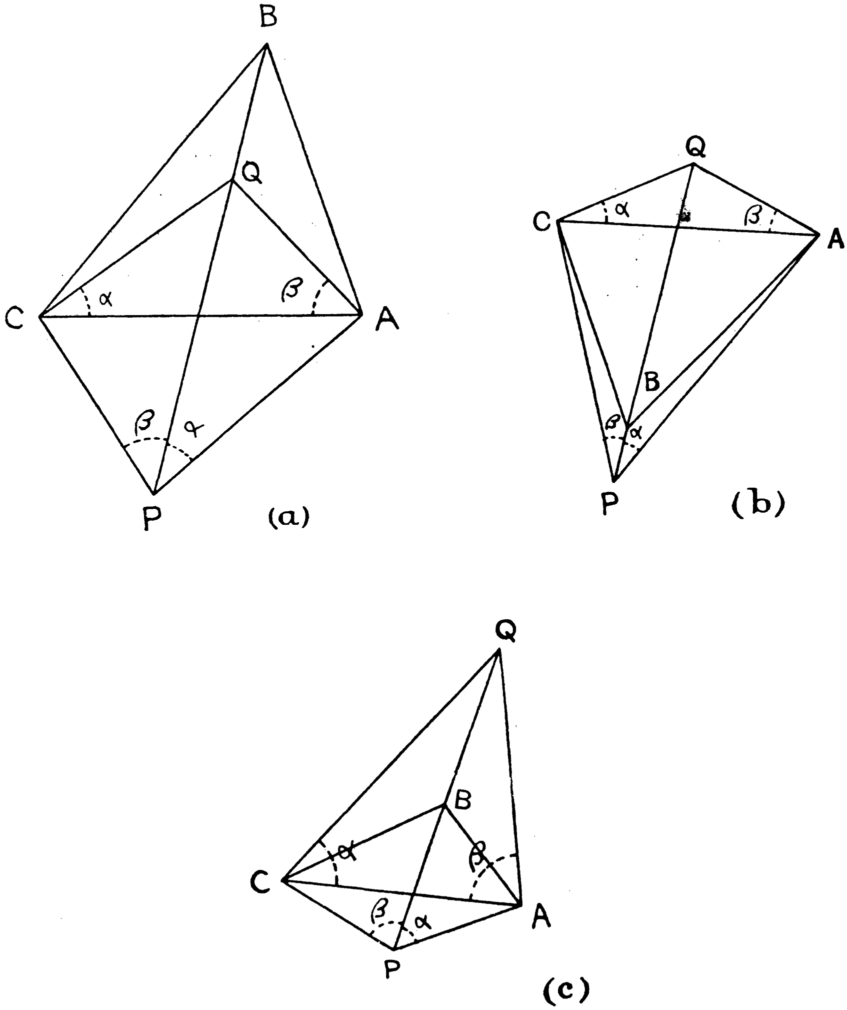


FIG. 30.

Example. Method II (Circle Method).

	Trig. point.	Observed angles.			Co-ordinates. East.	Co-ordinates. North.				
		°	′	″				°	′	″
A	*Beacon Tumulus TP*	*00*	*00*	*00*	*461884·4*	*162687·0*	α	*40*	*51*	*31*
B	*Knighton TP*	*179*	*24*	*40*	*456688·4*	*165698·1*	β	*57*	*08*	*10*
C	*New Buildings TP*	*220*	*16*	*11*	*457842·0*	*167629·2*	λ	*82*	*35*	*39*
D	*Syrencot TP*	*277*	*24*	*21*	*460844·5*	*166790·0*				

B *456688·4* *165698·1*
C *457842·0* *167629·2*

1153·6 *1931·1*

Log. DE *3·0620552*
„ DN *3·2858048*

„ Tan θ *9·7762504*

θ *30° 51′ 12″*

Log. DE *3·0620552* Log. DN *3·2858048*
„ sin θ *9·7099838* „ cos θ *9·9337316*

3·3520714 *3·3520732*

Mean Distance BC = *3·3520723*

C *457842·0* *167629·2*
D *460844·5* *166790·0*

3002·5 *839·2*

Log. DE *3·4774830*
„ DN *2·9238655*

„ Tan θ *10·5536175*

θ *105° 36′ 57″*

Log. DE *3·4774830* Log. DN *2·9238655*
„ sin θ *9·9836661* „ cos θ *9·4300524*

3·4938169 *3·4938131*

Mean Distance CD = *3·4938150*

D *460844·5* *166790·0*
A *461884·4* *162687·0*

1039·9 *4103·0*

Log. DE *3·0169916*
„ DN *3·6131015*

„ Tan θ *9·4038901*

θ *165° 46′ 41″*

Log. DE *3·0169916* Log. DN *3·6131015*
„ sin θ *9·3903674* „ cos θ *9·9864812*

3·6266242 *3·6266203*

Mean Distance DA = *3·6266222*

To find Distance CR.

Log. Dis. CB *3·3520723*
Co-log. 2 *1̄·6989700*
Cosec α *0·1842930*

Distance CR = *3·2353353*

To find Distance CS.

Log. Dis. CD *3·4938150*
Co-log. 2 *1̄·6989700*
Cosec β *0·0757403*

Distance CS = *3·2685253*

To find Distance DT.

Log. Dis. DA *3·6266222*
Co-log. 2 *1̄·6989700*
Cosec λ *0·0036381*

Distance DT = *3·3292303*

To find Bearing CR. ° ′ ″

Bearing CB *210 51 12*
90 — α *49 08 29*

Bearing CR = *161 42 43*

To find Bearing CS. ° ′ ″

Bearing CD *105 36 57*
90 — β *32 51 50*

Bearing CS *138 28 47*

To find Bearing DT. ° ′ ″

Bearing DA *165 46 41*
90 — λ *+07 24 21*

Bearing DT *173 11 02*

Point.	Bearing.	Computation.	Log. Distance.	Anti-log. E.	Anti-log. N.	Co-ordinates of R. S. and T.	E.	N
C to R	*161° 42′ 43″*	Sin *9·4966454* Dis *3·2353353* Cos *9·9774908*	*2·7319807* *3·2128261*	*539·49* +	*1632·4* —	C R	*457842·0* *+539·49* *458381·49*	*167629·2* *—1632·4* *165996·8*
C to S	*138° 28′ 47″*	Sin *9·8214382* Dis *3·2685253* Cos *9·8743201*	*3·0899635* *3·1428454*	*1230·2* +	*1389·5* —	C S	*457842·0* *+1230·2* *459072·2*	*167629·2* *—1389·5* *166239·7*
D to T	*173° 11′ 02″*	Sin *9·0743892* Dis *3·3292303* Cos *9·9969196*	*2·4036195* *3·3261499*	*253·29* +	*2119·1* —	D T	*460844·5* *+253·29* *461097·79*	*166790·0* *—2119·1* *164670·9*

Co-ords. R = *458381·49* *165996·8*
„ S = *459072·2* *166239·7*

690·71 *242·9*

Log. DE = *2·8392957*
„ DN = *2·3854275*

Log. Tan θ *10·4538682*

θ *70° 37′ 30″*

Co-ords. S = *459072·2* *166239·7*
Co-ords. T = *461097·79* *164670·9*

2025·59 *1568·8*

Log. DE = *3·3065516*
„ DN = *3·1955676*

Log. Tan θ *10·1109840*

θ *127° 45′ 27″*

To find Bearing RP. ° ′ ″

Bearing RC *341 42 43*
„ RS *430 37 30*

88 54 47
„ RS *70 37 30*

„ RP *159 32 17*

To find Bearing SP. ° ′ ″

Bearing SC *318 28 47*
„ SR *250 37 30*

67 51 17
„ SR *250 37 30*

„ SP *182 46 13*

To find Bearing TP. ° ′ ″

Bearing TP *353 11 02*
„ TS *307 45 27*

45 25 35
„ TS *307 45 27*

„ TP *262 19 52*

Point.	Bearing.	Computation.	Log. Dis.	Anti-Log. East.	Anti-Log. North.	Co-ordinates.	East.	North.
	° ′ ″							
R to P	*159 32 17*	Sin *9·5435530* Dis *3·2353353* Cos *9·9716954*	*2·7788883* *3·2070307*	*601·02* +	*1610·8* —	R P	*458381·49* *+601·02* *458982·51*	*165996·8* *—1610·8* *164386·0*
S to P	*182 46 13*	Sin *8·6842315* Dis *3·2685253* Cos *9·9994922*	*1·9527568* *3·2680175*	*89·693* —	*1853·6* —	S P	*459072·2* *—89·69* *458982·51*	*166239·7* *—1853·6* *164386·1*
T to P	*262 19 52*	Sin *9·9960981* Dis *3·3292303* Cos *9·1253123*	*3·3253284* *2·4545426*	*2115·1* —	*284·8* —	T P	*461097·79* *—2115·1* *458982·69*	*164671·0* *—284·8* *164386·2*

2. Method II Fig. 31 (Circle method).

Theory. Referring to Fig. 31, if A, B, C and D be four trig points from which a point P is to be resected, and if α, β and λ be the angles $B\hat{P}C$, $C\hat{P}D$, $D\hat{P}A$.

Then the circle CBP has as its centre R, which is on the perpendicular bisector of BC, and is such a point that $B\hat{R}C = 2\ B\hat{P}C = 2\alpha$. Similarly P lies on circle centres S and T. P is therefore the inter-section of these 3 circles.

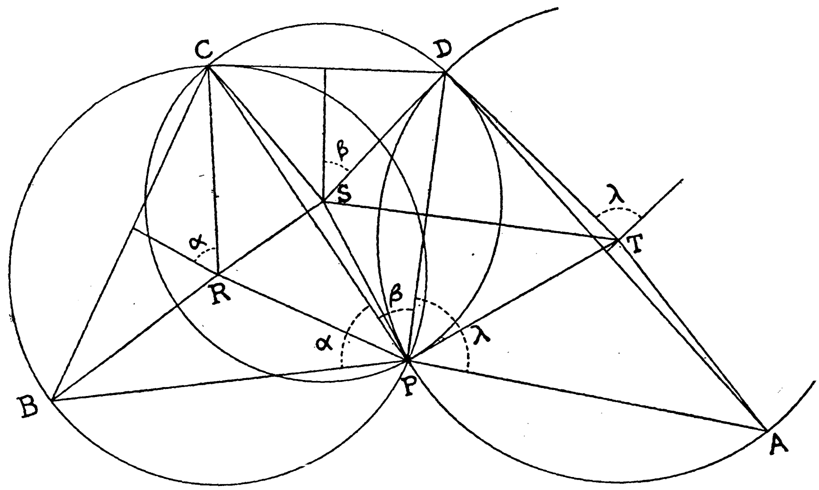


FIG. 31.

Procedure. The distances BC, CD, DA are computed, and from the relation $CR = \frac{CB}{2} \times \operatorname{cosec} \alpha$, etc., we compute CR, CS and DT. From the relative bearing CR = bearing CB — $(90° - \alpha)$, etc. we get the bearings CR, CS and DT. Hence we can find the co-ordinates of R, S and T.

The bearings RS and ST can then be computed. Now the triangles SRC and SRP are similar and equal. Hence bearing RP = bearing RS + $P\hat{R}S$ = bearing RS + $C\hat{R}S$ = bearing RS + bearing RS — bearing RC. Similarly for bearings SP and TP.

Now we also know the distance RP (which equals RC) etc. Hence we can compute the co-ordinates of P from R, S and T.

EXAMPLE OF COMPUTATION OF SEMI-GRAPHIC RESECTION.

(Reverse Bearing Method.)

OBSERVATIONS AT THE RESECTED POINT. REDUCED ANGLES.

Trigonometrical Point.		Observed Angles.			Co-ordinates of Trigonometrical Points. East.	Co-ordinates of Trigonometrical Points. North.	Approximate Distances. Metres.
		°	′	″			
A	Beacon Tumulus TP	*00*	*00*	*00*	*461884·4*	*162687·0*	*3350*
B	Pauper's Prospect TP	*39*	*34*	*06*	*460158·6*	*161169·3*	*3400*
C	King's Barrow TP	*90*	*07*	*26*	*457409·8*	*161713·4*	*3100*
D	Knighton TP	*179*	*24*	*40*	*456688·4*	*165698·1*	*2650*
E	Syrencot TP	*277*	*24*	*21*	*460844·5*	*166790·0*	*3050*
	Trial Point				*458980·0*	*164390·0*	

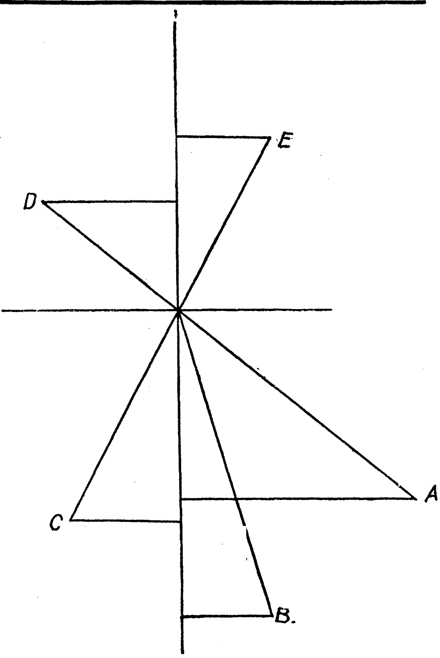
—	Co-ordinates. East.	Co-ordinates. North.
Trial Point	*458980·0*	*164390·0*
A	*461884·4*	*162687·0*
Differences	*2904·4*	*1703·0*

Log Diff. Easting *3·4630564*
Log Diff. Northing *3·2312146*

Log Tan Bearing *10·2318418*

Bearing *120° 23′ 07″*

Bearings Trial Point to :—		Reverse Bearings
A	*120° 23′ 07″*	*300° 23′ 07″*
B	*159 57 13*	*339 57 13*
C	*210 30 33*	*30 30 33*
D	*299 47 47*	*119 47 47*
E	*37 47 28*	*217 47 28*



Computation of Cutting Points on North and South Grid Line.

—	B.	C.	D.	E.
Easting of Trig Point	*460158·6*	*457409·8*	*456688·4*	*460844·5*
Easting of Trial Point	*458980·0*	*458980·0*	*458980·0*	*458980·0*
Difference	*1178·6*	*1570·2*	*2291·6*	*1864·5*
Log Difference	*3·0713664*	*3·1959550*	*3·3601388*	*3·2705624*
Log Cotan Bearing	*10·4378411*	*10·2296926*	*9·7578678*	*10·1104568*

Trigonometrical Point.		Observed Angles.			Co-ordinates of Trigonometrical Points.		Approximate Distances.
					East.	North.	Metres.
		°	′	″			
A	Beacon Tumulus TP	*00*	*00*	*00*	*461884·4*	*162687·0*	*3350*
B	Pauper's Prospect TP	*39*	*34*	*06*	*460158·6*	*161169·3*	*3400*
C	King's Barrow TP	*90*	*07*	*26*	*457409·8*	*161713·4*	*3100*
D	Knighton TP	*179*	*24*	*40*	*456688·4*	*165698·1*	*2650*
E	Syrencot TP	*277*	*24*	*21*	*460844·5*	*166790·0*	*3050*
	Trial Point				*458980·0*	*164390·0*	

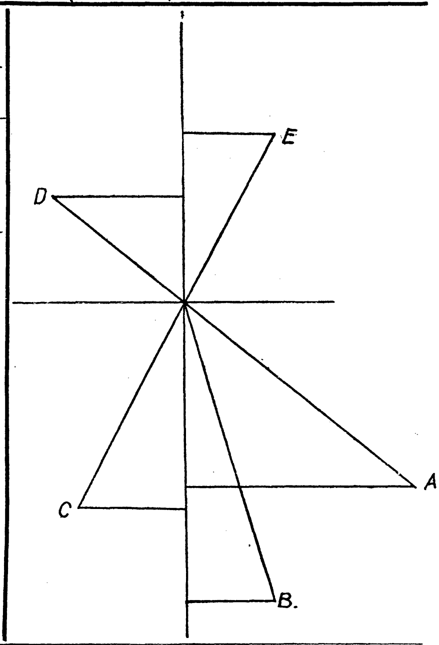
—	Co-ordinates.	
	East.	North.
Trial Point	*458980·0*	*164390·0*
A	*461884·4*	*162687·0*
Differences	*2904·4*	*1703·0*

Log Diff. Easting *3·4630564*
Log Diff. Northing *3·2312146*

Log Tan Bearing *10·2318418*

Bearing *120° 23′ 07″*

Bearings Trial Point to :—				Reverse Bearings		
A	*120°*	*23′*	*07″*	*300°*	*23′*	*07″*
B	*159*	*57*	*13*	*339*	*57*	*13*
C	*210*	*30*	*33*	*30*	*30*	*33*
D	*299*	*47*	*47*	*119*	*47*	*47*
E	*37*	*47*	*28*	*217*	*47*	*28*



Computation of Cutting Points on North and South Grid Line.

—	B.	C.	D.	E.
Easting of Trig Point	*460158·6*	*457409·8*	*456688·4*	*460844·5*
Easting of Trial Point	*458980·0*	*458980·0*	*458980·0*	*458980·0*
Difference	*1178·6*	*1570·2*	*2291·6*	*1864·5*
Log Difference	*3·0713664*	*3·1959550*	*3·3601388*	*3·2705624*
Log Cotan Bearing	*10·4378411*	*10·2296926*	*9·7578678*	*10·1104568*
Log Distance	*3·5092075*	*3·4256476*	*3·1180066*	*3·3810192*
Distance	*3230·0*	*2664·7*	*1312·2*	*2404·5*
Northing of Trig Point	*161169·3*	*161713·4*	*165698·1*	*166790·0*
Northing of Cutting Pt.	***164399·3***	***164378·1***	***164385·9***	***164385·5***

Example of Computation of Semi-Graphic Resection.

(Reverse Bearing Method.)

Observations at the Resected Point. Reduced Angles.

Trigonometrical Point.		Observed Angles. ° ′ ″	Co-ordinates of Trigonometrical Points. East.	North.	Approximate Distances. Metres.
A	Beacon Tumulus TP	00 00 00	461884·4	162687·0	3350
B	Pauper's Prospect TP	39 34 06	460158·6	161169·3	3400
C	King's Barrow TP	90 07 26	457409·8	161713·4	3100
D	Knighton TP	179 24 40	456688·4	165698·1	2650
E	Syrencot TP	277 24 21	460844·5	166790·0	3050
	Trial Point		458980·0	164390·0	

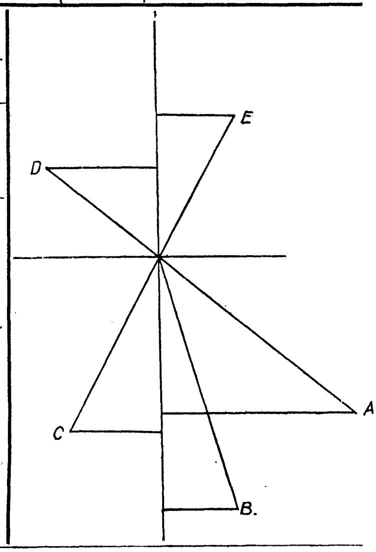
	Co-ordinates. East.	North.
Trial Point	458980·0	164390·0
A	461884·4	162687·0
Differences	2904·4	1703·0

Log Diff. Easting 3·4630564
Log Diff. Northing 3·2312146

Log Tan Bearing 10·2318418

Bearing 120° 23′ 07″

Bearings Trial Point to :—		Reverse Bearings
A	120° 23′ 07″	300° 23′ 07″
B	159 57 13	339 57 13
C	210 30 33	30 30 33
D	299 47 47	119 47 47
E	37 47 28	217 47 28



Computation of Cutting Points on North and South Grid Line.

—	B.	C.	D.	E.
Easting of Trig Point	460158·6	457409·8	456688·4	460844·5
Easting of Trial Point	458980·0	458980·0	458980·0	458980·0
Difference	1178·6	1570·2	2291·6	1864·5
Log Difference	3·0713664	3·1959550	3·3601388	3·2705624
Log Cotan Bearing	10·4378411	10·2296926	9·7578678	10·1104568
Log Distance	3·5092075	3·4256476	3·1180066	3·3810192
Distance	3230·0	2664·7	1312·2	2404·5
Northing of Trig Point	161169·3	161713·4	165698·1	166790·0
Northing of Cutting Pt.	164399·3	164378·1	164385·9	164385·5

35. Semi-graphic Methods.

1. *Reverse bearing method* (*Semi-graphic*). Fig. 32.

Let A, B, C, D etc. be trig points to which observations have been taken from a point T (Fig. 32).

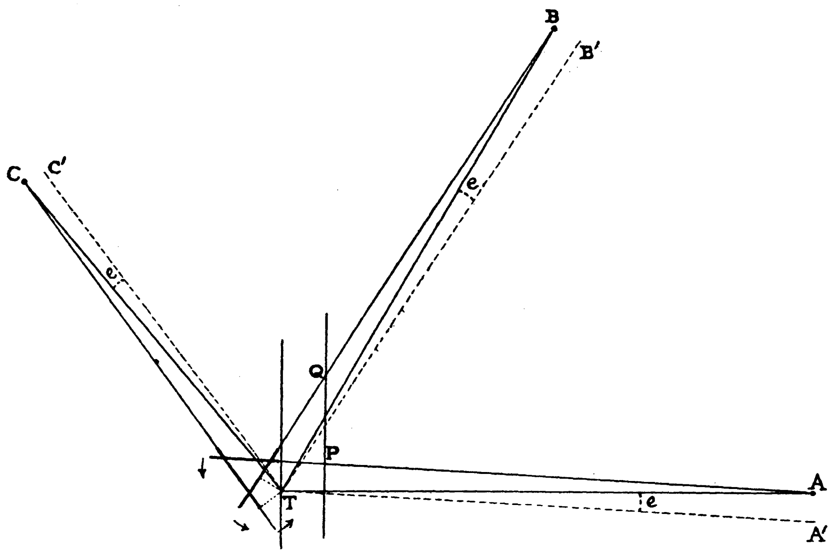


Fig. 32.

An approximate position of T, or "trial point," P, is first obtained by plane table or some other graphic method.

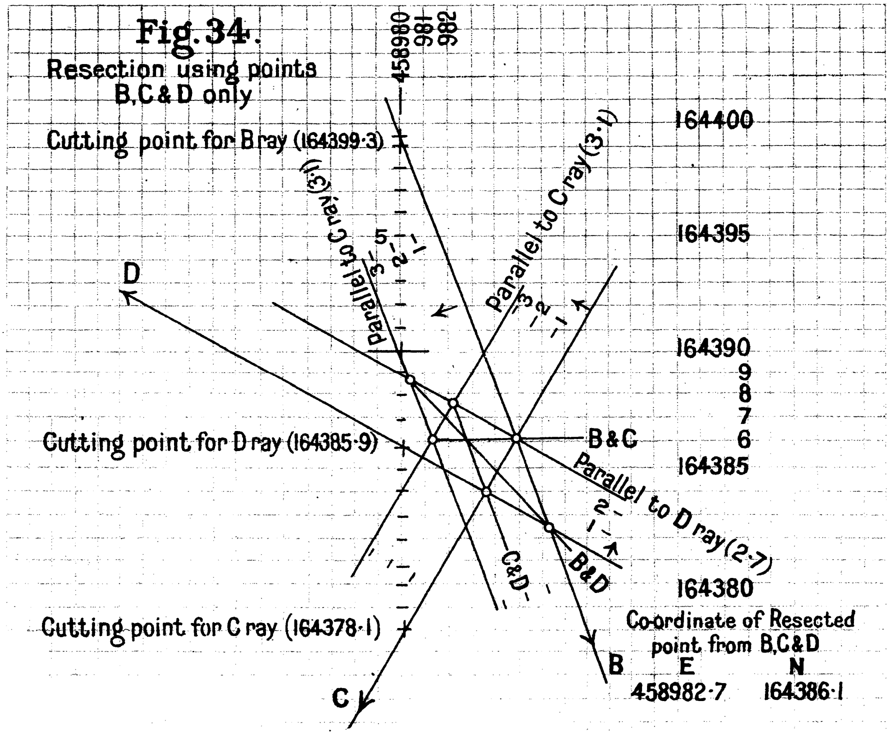
From the co-ordinates of P, and of one, preferably the most distant, trig point (say A) the bearing PA is calculated.

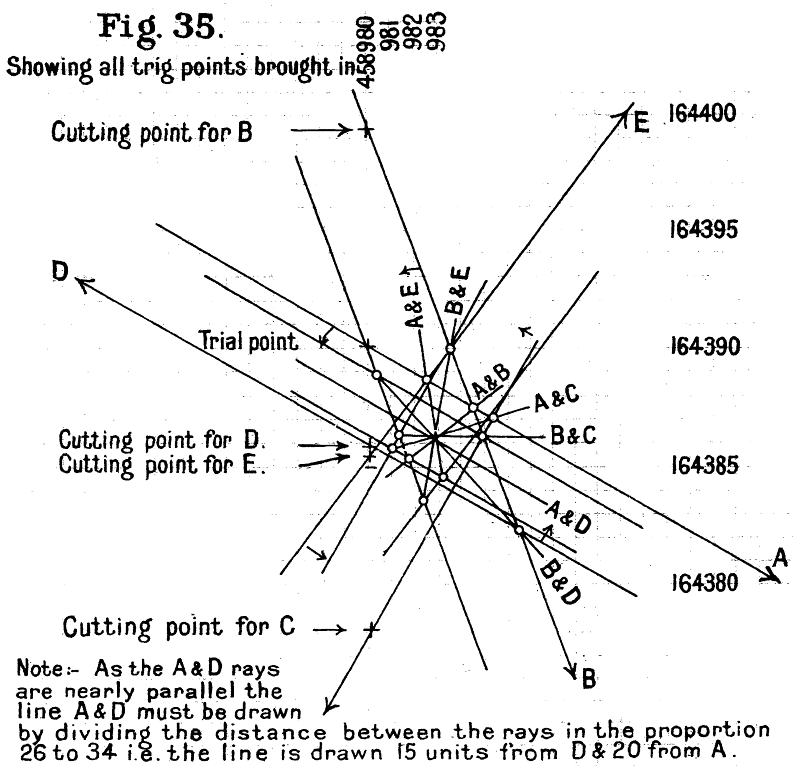
To this approximate bearing the observed angles are added (or subtracted as the case may be) to obtain values of the bearings to the other trig points.

If P coincided exactly with T, these values would be the actual bearings TA, TB, etc., and since the bearing from one trig point to another can be calculated, all the angles of the triangles TAB, TBC, etc., could be deduced, and the position of T obtained directly from their solution.

Since, however, the co-ordinates of P will probably not be those of the point from which the angles were actually observed, the bearing PA will differ from TA by a small amount *e*. Consequently, the deduced values of the bearings TB, TC, etc., obtained by adding the observed angles, will be all in error by the same amount *e*, and, if drawn out from the "true" position T, would be represented by the rays TA′ TB′ etc.

If each of these bearings, instead of being drawn through T, is drawn through the appropriate trig point, then, considering at first only three points A, B and C, it can be seen that they will not meet in a point, but form a "triangle of error," from which the position of T must be deduced by means of the rules stated in Section 32.





Malby & Sons. Lith.

The derivation of these rules can be seen from the figure, *e.g.* :—

In order that the rays may meet in a point they must all swing in the same sense through the same angle.

The perpendicular distance of the point T from each ray is TA sin *e*, TB sin *e*, TC sin *e*, etc., and is consequently proportional to the distance from A, B, C, &c.

As P is by hypothesis close to T compared with the lengths TA, TB, &c., it is sufficiently correct to say that these perpendicular distances are proportional to PA, PB, &c.

The position of P *can consequently be deduced from the triangle of error.* The triangle of error for each triangle can be drawn out on a suitable scale by the following device. In Fig. 33 let P and B be the trial point and trig point respectively.

The bearing QB is known, and so is the distance PN, for it is the difference between the E co-ordinates of B and P.

BM can therefore be calculated.

BN is the difference between the N co-ordinates of B and P, and therefore PQ=NM

=BN—BM.

Thus, PQ can be calculated, and the ray in the vicinity of P drawn in by ruling two lines at right angles through P to represent PY and PN, scaling off PQ to any suitable scale along PY, and at Q laying off the angle YQB with a protractor.

The point Q is called a " cutting point " of the reverse bearing from B along the N and S grid line.

It is to be noted that the cutting point R on the E and W grid line can be calculated in an analogous manner, and, if a protractor is not at hand, the ray QB might be drawn in by doing so and by joining Q and R.

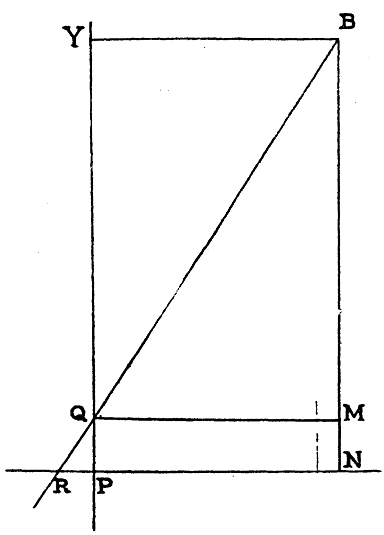


FIG. 33.

In the same manner other cutting points on one or other, or both, of the grid lines can be calculated from the remaining trig points, and all the rays can be plotted on the same graph without making use of the points A, B, C, &c., in the drawing.

The drawing can be done on any convenient scale.

The use of squared paper expedites the drawing in many ways.

Fig. 34 represents the diagram or graph at this stage.

Having thus obtained the triangle of error the next step is to interpolate the required position of T.

This may be done as follows :—

Note, by inspection, in which direction the rays have to swing to bring them to a point, and on the side to which this swing is made, draw, parallel to each ray, a straight line at a distance from the ray proportional to the distance of P from the trig point.

(The exact distance is immaterial as long as the proportion of one distance to another is maintained correct.)

This is conveniently done by scaling off PA, PB, &c., from a map or chart in hundreds or thousands of metres (or yards or feet as the case may be), drawing perpendiculars (by eye) to XA, XB, &c., representing these lengths on any suitable scale, and drawing lines through the end of each perpendicular parallel to the corresponding ray.

If the two rays meet in X, and their parallels meet in X^1, then the true position of T lies on XX^1.

In this way a line passing through T is obtained from each pair rays, and the position of T is given by the intersection of these lines.*

The difference between the E and N co-ordinates of T and P can be scaled off from the graph, and applied as corrections to the already known co-ordinates of P, or if squared paper is used, the lines on the squared paper can be numbered to correspond with the grid lines in the vicinity of P, and the co-ordinates of T read off directly from the graph.

* The proof of this construction is as follows :—

Figure 35 shows the continuation of the graph given in figure 34 to determine the position of T.

Let P be the trial point (Fig. 35A).

T the true position of the resected point.

A and B two trigonometrical points.

Then if PA is the *calculated* bearing to A and TA is the *true* bearing. The reverse bearing laid out on the graph is represented by AP and is too small by the angle PAT.

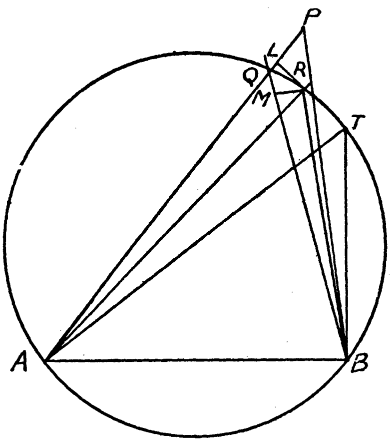


FIG. 35A.

Now the reverse bearing from B is obtained by subtracting the observed angle ATB from the calculated bearing PA and adding 180. The bearing obtained by subtracting the angle ATB from PA is bearing QB, if AQB is made equal to ATB.

But if AQB = ATB, Q must be a point on the circle circumscribing A, B and T.

BQ is, however, the reverse bearing from B as laid out on the graph. Q its intersection with PA is therefore a point on the circle circumscribing A, B and T.

Consider now a point R such that RL, perpendicular to AP, and RM, perpendicular to BP, are proportional respectively to the lengths RA and RB, so that :—

RL : RM :: RA : RB

The right-angled triangles ARL and BRM are therefore similar and therefore $L\hat{A}R = M\hat{B}R$. But if $L\hat{A}R = M\hat{B}R$, A, B, Q and R must lie on the circumference of a circle.

R must therefore be another point on the circle circumscribing ABT and an arc drawn through Q and R will pass through T. If Q, R and T are all close together in comparison with the distance away of A and B, this arc can be represented without appreciable error by a straight line joining Q and R.

In using this construction it may happen that the rays AP and BQ on the graph are nearly parallel and their intersection comes off the paper. In such cases to draw in the arc it is necessary to construct two points on the circle in a similar way to that described above and join them.

2. *The Tangents method* (Fig. 36).

The theory of this method is as follows :—

Let P be the trial point, and A and B two of the trig points, and let XPY be a tangent to the circle circumscribing P, A and B.

Join PA and PB, and let TRAB be a circle passing through A, B and the true position of the resected point.

Then from the geometry of the figure since XY is a tangent at P to the circle PAB.

$$\angle XPA = \angle ABP$$

And since ATRB is a quadrilateral inscribed in a circle

$$\angle ATR = 180° - \angle ABP$$

$$\therefore \angle PTR = \angle ABP$$
$$= \angle XPA.$$

Hence XY is a parallel to TR.
Draw PS perpendicular to TR

Then

$$PS = PT \sin PTR = PT \sin ABP.$$

But from the triangle PTB $PT = \dfrac{BT \sin TBP}{\sin TPB}$,

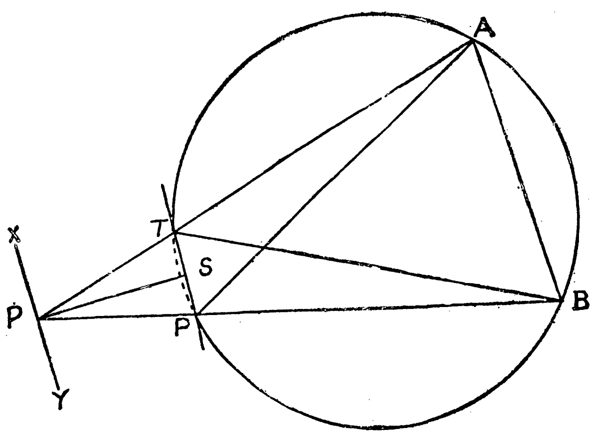


Fig. 36.

and from the Δ ABP, $\sin ABP = \frac{PA \sin APB}{AB}$.

$$\text{Hence } PS = \frac{BT \sin TBP}{\sin TPB} \times \frac{PA \sin APB}{AB}$$

$$= \frac{BT \times PA}{AB} \sin TBP = \frac{BT \times PA}{AB} \sin (ATB - APB)$$

But, since PT is by hypothesis small compared to PA, or BP, we may say BT = BP approximately, and

$$PS = \frac{PB \times PA}{AB} \sin (ATB - APB).$$

Now, if the bearings PA and PB are computed from the co-ordinates of P and the two trig points

$\angle$ APB = Bearing PB — Bearing PA.

This is called the *computed angle* at P.

And, since the true position of the resected point lies, by hypothesis, on the circle TRAB,

$\angle$ ATB = *observed angle* at the resected point.

These two angles, computed and observed, are usually written C and O respectively.

$$\text{So that } PS = \frac{PA \times PB}{AB} \sin (O - C).$$

Again, since P is supposed to be close to the true position O — C is a small angle, therefore—

$$\sin (O - C) = (O - C) \text{ radians}$$

$$= \frac{\pi}{180 \times 60} (O - C) \text{ minutes.}$$

If PA, PB and AB are taken as thousands of units (yards, metres or feet) we have :—

$$PS \text{ (in the same units)} = \frac{PA \times 1000 \times PB \times 1000}{AB \times 1000} \times \frac{\pi}{10800} \times (O - C)$$

$$= \frac{3}{10} \frac{PA \times PB}{AB} \times (O - C)$$

very nearly.

EXAMPLE: THE TANGENTS METHOD.

—	A.		B.		C.		D.	
Co-ordinates of Trig Pts. Co-ordinates of Trial Point	461884·4 458980·0	162687·0 164390·0	460158·6 458980·0	161169·3 164390·0	457409·8 458980·0	161713·4 164390·0	456688·4 458980·0	165698·1 164390·0
	2904·4	1703	1178·6	3220·7	1570·2	2676·6	2291·6	1308·1
Log diff. Easting ... Log diff. Northing ...	3·4630564 3·2312146		3·0713664 3·5079503		3·1959550 3·4275835		3·3601388 3·1166077	
Log bearing	0·2318418		$\bar{1}$·5634161		$\bar{1}$·7683715		0·2435311	
Bearing	120° 23′ 07″		159° 54′ 00″		210° 23′ 51″		299° 43′ 00″	

—	A.	B.	C.		D.		
		B − A	C − A	C − B	D − A	D − B	D − C
Computed angle ... ... Observed angle		39° 30′ 53″ 39° 34′ 06″	90° 00′ 44″ 90° 07′ 26″	50° 29′ 51″ 50° 33′ 20″	179° 19′ 53″ 179° 24′ 40″	139° 49′ 00″ 139° 50′ 34″	89° 19′ 09″ 89° 17′ 14″
O − C		+ 3′ 13″ + 3·2′	+ 6′ 42″ + 6·7′	+ 3′ 29″ + 3·5′	+ 4′ 47″ + 4·8′	+ 1′ 34″ + 1·6′	− 1′ 55″ − 1·9′
	Formula— $\frac{3}{10}\cdot\frac{PA.PB}{AB}\times(o-c)$ (By slide rule)	$\frac{3}{10}\times\frac{3\cdot4\times3\cdot4}{2\cdot3}\times3\cdot2$ $\frac{9\cdot6\times11\cdot6}{23}$ + 4·85	$\frac{3}{10}\cdot\frac{3\cdot4\times3\cdot1}{4\cdot6}\times6\cdot7$ $\frac{20\cdot1\times10\cdot5}{46}$ + 4·55	$\frac{3}{10}\times\frac{3\cdot4\times3\cdot1}{2\cdot8}\times3\cdot5$ $\frac{10\cdot5\times10\cdot5}{28}$ + 3·9	$\frac{3}{10}\cdot\frac{3\cdot4\times2\cdot6}{6\cdot0}\times48$ $\frac{2\cdot4\times8\cdot8}{10}$ + 2·1	$\frac{3}{10}\cdot\frac{3\cdot4\times2\cdot6}{5\cdot7}\times1\cdot6$ $\frac{4\cdot8\times8\cdot8}{57}$ + ·74	$\frac{3}{10}\cdot\frac{2\cdot6\times3\cdot1}{4\cdot0}\times1\cdot9$ $\frac{5\cdot7\times8\cdot0}{40}$ − 1·1
Opposite angle (measured by protractor) ... Bearing of Trig Point		PAB 71° 30′ 159° 54′	PAC 42° 15′ 210° 24′	PBC 58° 10′ 210° 24′	PAD 0° 15′ 299° 43′	PBD 17° 10′ 299° 43′	PCD 41° 00′ 299° 43′
Bearing of Tangent		231° 24′	252° 39′	268° 34′	299° 58′	316° 53′	340° 43′

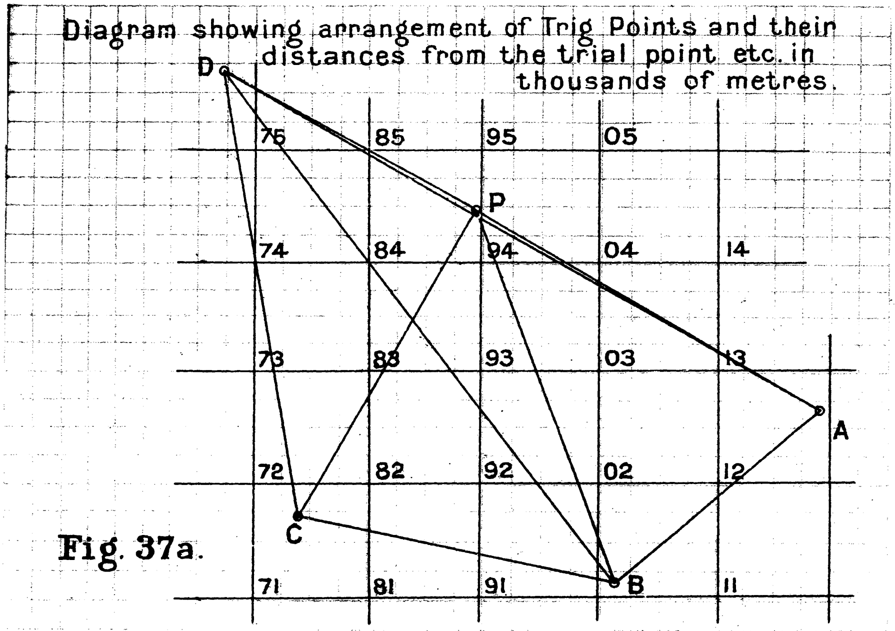


Fig. 37a.

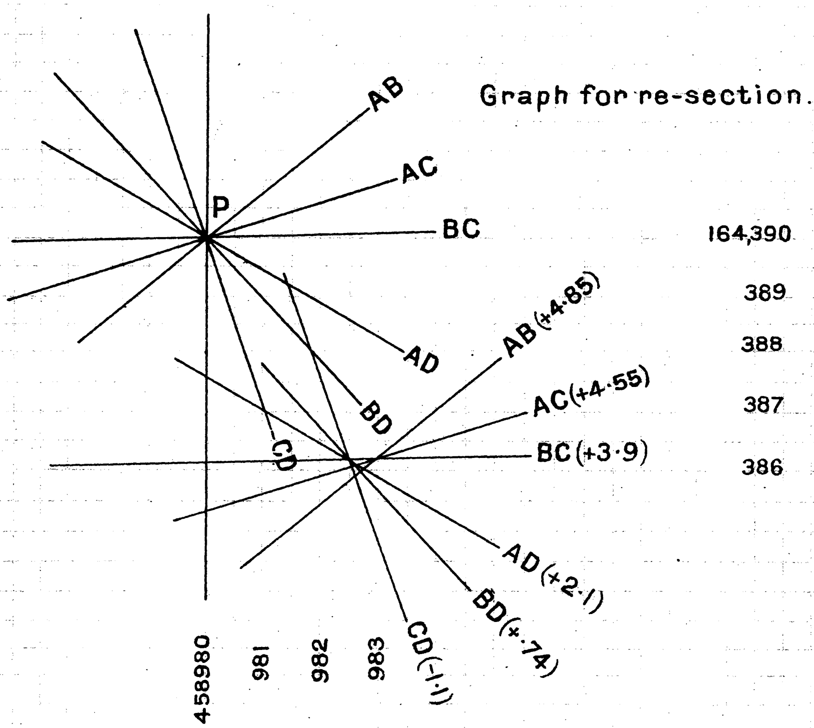


Fig. 37b.

Co-ordinates of point selected
458982·8, 164386·2

Malby & Sons, Lith.

Note that if O — C is positive, *i.e.*, if O is greater than C, the line TR, which for practical purposes coincides with the arc of the circle on which the true position lies, lies between P and two trig points. If O — C is negative, the true position is on the opposite side of P to the two trig points.

These principles are made use of thus :—

From the co-ordinates of P the bearings to all trig points are computed to give the values of C for each pair of trig points.

These are compared with the corresponding observed angles to get the values of O — C.

The lengths PA, PB, &c., AB, AC, BC, &c., are scaled off from the map, and the angles ABP, BCP, ACP, &c., measured with a protractor.

The bearings of the tangents corresponding to XY for each pair of trigs are obtained by adding or subtracting, as the case may be, the angles ABP, BCP, &c., to or from the calculated bearings PA, PB, &c.

A graph is prepared with a "trial point" as centre by laying out the various tangents with a protractor, and drawing lines parallel to them at distances from them obtained from the formulæ—

$\frac{3}{10}\frac{PA \times PB}{AB} \times (O - C)$, which is conveniently worked out by slide-rule.

An example of a resection by this method is given below using the same data as in reverse bearing method (*see* Fig. 37 (*a*) and (*b*)).

36. The Logarithmic Method.

The theory of this method is as follows :—

Suppose P be the trail point and A, B, C, &c., trig points. Denote the co-ordinates of P, A, B, &c., by E_P, E_A, E_B and N_P, N_A, N_B, &c., and the bearings PA, PB, &c., by θ_A, θ_B, &c.

Then we have—

$$\tan \theta_A = \frac{E_P - E_A}{N_P - N_A} = \frac{D_E}{D_N} \text{ say} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Now consider a point close to P whose co-ordinates differ from those of P by the small quantities e and n.

The bearing to A from this point will differ from PA by a small quantity which we will call t_A.

We have then for this point—

$$\tan (\theta_A + t_A) = \frac{E_P + e - E_A}{N \ + n - N_A} = \frac{D_E + e}{D_N + n} \ldots\ldots\ldots \quad (ii)$$

Taking logarithms of (i) and (ii) we have—

$$\log \tan \theta_A = \log D_E - \log D_N \quad \text{.....................} \quad \text{(iii)}$$
$$\log \tan (\theta_A + t_A) = \log (D_E + e) - \log (D_N + n) \quad \text{......} \quad \text{(iv)}$$

Now consider $\log (D_E + e)$.

If we put $\log D_E + 1 - \log D_E = A_E$ ....

so that A_E is the change in the logarithm of D_E caused by adding one *unit* to it, it will be found that as long as e is small compared to E_P and E_A :—

$$\log (D_E + e) = D_E + e \,.\, A_E.$$

Similarly using the same notation

$$\log (D_N + n) = \log D_N + n \,.\, A_N$$
$$\text{and } \log \tan (\theta_A + t_A) = \log \tan \theta_A + t_A \,.\, A_T.$$

Equation (iv) can then be written

$$\log \tan \theta_A + t_A \,.\, A_T = \log D_E + e \,.\, A_E - \log D_N - n \,.\, A_N \quad \text{(v)}$$

Subtracting (i) from (v) we have

$$t_A \,.\, A_T = e \,.\, A_E - n \,.\, A_N \quad \text{......................................} \quad \text{(vi)}$$

In this equation it is assumed that t, e and n, may have any sign, but the equation will always be true as long as these proper signs are inserted.

The signs of each term, however, depend also on those of A_T, A_E and A_N which must now be considered.

Consider first the term $e \,.\, A_E$. This represents the quantity

$$\log (D_E + e) - \log D_E.$$

Assuming, as will always be the case, that E_A, E_P, N_A, N_P, &c., are positive quantities.

Then, if D_E is positive, i.e., E_P greater than E_A and e is also positive, $D_E + e$ will be *numerically* greater than D_E.

The value of the logarithm depends on the numerical values of D_E, hence the term $e \,.\, A_E$ will be positive; but it has been assumed that e is positive, therefore A_E must also be positive, or the product eA_E would be negative.

Now, suppose D_E is still positive but e is negative

$(D_E + e)$ will be numerically less than D_E, consequently $\log (D_E + e) - \log D_E$, *i.e.*, eA_E, will be negative,

but, as e is assumed negative, A_E must be positive or the product eA_E would become positive.

Hence whatever the sign of e, A_E is positive when D_E is positive. *Again, if D_E is negative, i.e.,* if E_P is less than E_A.

If e is positive, then *numerically* $D_E + e$ will be less than D_E . $\log D_E + e$ will be less than D_E and eA_E will be negative. But e is assumed positive, so that A_E must be negative or the product would be positive also; again, if e is negative and D_E also negative, $D_E + e$ will be numerically greater than D_E and eA_E will be positive, but e is assumed negative, therefore A_E must also be negative or the product eA_E would be negative. Hence, whatever the sign of e, A_E is negative when D_E is negative.

Exactly the same reasoning applies to A_N and we deduce that whatever the signs of e and n the signs of Ae and An are the same as those of D_E and D_N.

Now consider the term t_A A_T.

t_A represents a change in the angle θ_A; A_T a change in logarithm of the tangent of it.

If the angle increases, it does not follow that the numerical value of the tangent also increases. It only does so in fact in the first and third quadrants when the tangent is positive. *In the first and third quadrants when tan* θ_A *is positive* if t_A is positive the angle $\theta_A + t_A$ is greater than θ_A and log tan $(\theta_A + t_A)$ is greater than log tan θ_A.

$t_A . A_T$, which is log tan $(\theta_A + t_A)$ — log tan θ_A, is therefore positive, but t_A is assumed positive, therefore A_T must also be positive.

Similarly in these quadrants if t_A is negative both angle and tangent are diminished and log tan θ_A is greater than log tan $(\theta_A + t_A)$.

t_A . A_T is therefore negative, but t_A is *assumed* negative, therefore A_T . A_T must be positive.

In these quadrants when tan θ_A is positive A_T is also positive, whatever the sign of t_A.

In the second and fourth quadrants when tan θ_A *is negative if* t_A is positive the angle $\theta_A + t_A$ is greater than θ_A, but the tangent of $\theta_A + t_A$, though *algebraically* greater, is *numerically* less than tan θ_A. Hence $t_A A_T$ is negative, but t_A is assumed positive, therefore A_T must be negative.

Similarly, if t_A is negative the angle is diminished but the tangent numerically increased, so that $t_A A_T$ is positive; but t_A being negative, A_T must also be negative.

In these quadrants, when tan θ_A is negative, A_T is also negative.

Summing up then, we know the signs of D_E and D_N and therefore of tan θ_A and the signs of A_E, A_N and A_T are the same, respectively, as those of these quantities, whatever may be the signs of *e*, *n* and *t*.

Note however that D_E represents $E_P - E_A$ and not $E_A - E_P$.

Similarly with D_N.

Having thus determined the signs of A_E, A_N and A_T the equation (vi) can be written—

$$t_A = \frac{A_E}{A_T} e - \frac{A_N}{A_T} n$$

The quantities e and n being still indeterminate, both as to magnitude and sign.

In this equation t_A represents the difference between the bearings from a point P and from another point close to P, e and n away from it, to the trig point A.

In a similar way we may derive equations from the rays from the same two points to B, C and other trig points.

These equations may be written

$$t_B = \frac{B_E}{B_T} e - \frac{B_N}{B_T} n,$$

$$t_C = \frac{C_E}{C_T} e - \frac{C_N}{C_T} n,$$

and so on.

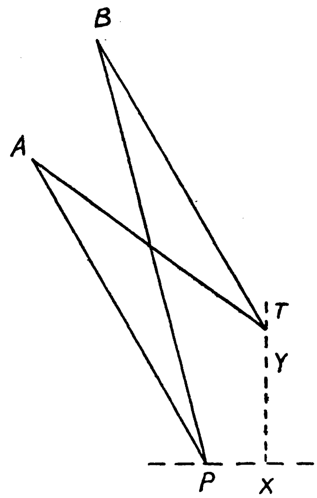


FIG. 38.

Now in Fig. 38, if we consider the true position of the resected point to be at T and separated from the trial point by distance e and n, we can obtain the values of e and n in the following manner :—

Conventionally, t_A represents TA — PA, for it is derived by subtracting equation (i) from equation (v).

Similarly, t_B represents TB — PB.

As long as the values and signs of e and n are unknown we cannot say if TA — PA or TB — PB should be negative or positive, but, if the points are taken in pairs in a clockwise rotation, it is possible to say that PB — PA and TB — TA are both positive. We have then—

$$t_A - t_B = TA - PA - (TB - PB)$$
$$= PB - PA - (TB - TA)$$

PB — PA is however the computed angle at P, obtained by calculating the bearings to A and B, using the co-ordinates of P, while TB — TA is the angle actually observed at the resected point.

Hence, using the same notation as in para. 2

$$t_A - t_B = C - O$$

or

$$e\left(\frac{A_E}{A_T} - \frac{B_E}{B_T}\right) - n\left(\frac{A_N}{A_T} - \frac{B_N}{B_T}\right) = C_{AB} - O_{AB} \ldots\ldots\ldots\ldots \quad \text{(vii)}$$

Similarly from the rays to A and C we have—

$$e\left(\frac{A_E}{A_T} - \frac{C_E}{C_T}\right) - n\left(\frac{A_N}{A_T} - \frac{C_N}{C_T}\right) = C_{AC} - O_{AC} \ldots\ldots\ldots\ldots \quad \text{(viii)}$$

Example—The Logarithmic Method.

—	A.		B.		C.		D.		E.	
Ep and Np ... EA and NA, &c. ...	458980·0 461884·4	164390·0 162687·0	458980·0 460158·6	164390·0 161169·3	458980·0 457409·8	164390·0 161713·4	458980·0 456688·4	164390·0 165698·0	458980·0 460844·5	164390·0 166790·0
	−2904·4	+1703·0	−1178·6	+3220·7	+1570·2	+2676·6	+2291·6	−1308·0	−1864·5	−2400·0
Log D_E ... ... Log D_N	3·4630564 3·2312146	$A_E=-1500$ $A_N=+2540$	3·0713664 3·5079503	$B_E=-3700$ $B_N=+1350$	3·1959550 3·4275835	$C_E=+2760$ $C_N=+1620$	3·3601388 3·1166077	$D_E=+1900$ $D_N=+3320$	3·2705624 3·3802112	$E_E=-2340$ $E_N=-1810$
Log tan Bearing ... Bearing	0·2318418 $A_T=-2895$ 120° 23′ 07″		$\bar{1}$·5634161 $B_T=-3927$ 159° 54′ 00″		$\bar{1}$·7683715 $C_T=+2895$ 210° 23′ 51″		0·2435311 $D_T=-2935$ 299° 43′ 00″		$\bar{1}$·8903512 $E_T=+2607$ 37° 50′ 34″	
C ... ... O	0 0 0 0 0 0		39° 30′ 53″ 39° 34′ 06″		90° 00′ 44″ 90° 07′ 26″		179° 19′ 53″ 179° 24′ 40″		277° 27′ 27″ 277° 24′ 21″	
C − O			AB − 3′ 13″		AC − 6′ 42″		− 04′ 47″		+ 3′ 06″	

—	A_E &c.	A_N &c.	A_T &c.	$\frac{A_E}{A_T}$ &c.	$\frac{A_N}{A_T}$ &c.	Observation Equations.
A	−15·0	+25·4	−29·0	+·52	−·87	$t_A-t_B=-\ \cdot43e-(-\ \cdot52n)=-3\cdot2$ or $+4\cdot3e-5\cdot2n=+32$
B	−37·0	+13·5	−39·3	+·95	−·35	$t_A-t_C=-\ \cdot43e-(-1\cdot42n)=-6\cdot7$ $\quad +4\cdot3e-14\cdot2n=+67$
C	+27·6	+16·2	+29·0	+·95	+·55	$t_A-t_D=+1\cdot17e-(-2\cdot01n)=-4\cdot8$ $\quad +11\cdot7e+20\cdot1n=-48$
D	+19·0	−33·2	−29·3	−·65	+1·14	$t_A-t_E=+1\cdot39e-(-\ \cdot19n)=+3\cdot1$ $\quad +13\cdot9e+1\cdot9n=+31$
E	−23·4	−18·1	−26·1	−·87	−·68	

Formation of Normal Equations—

e	n		e	n	
+ 18·5	− 22·3	+137·6	− 22·3	+ 27·0	−166
+ 18·5	− 61·0	+288·1	− 61·0	+202·0	−951
+137·0	+235·0	−562·0	+235·0	+404·	−965
+194·0	+ 26·4	+431	+ 26·4	+ 3·6	+ 59
+368	+177	+295	+177	+636	−2024

Normal Equations—

$$+3\cdot7e+1\cdot8n=+3\cdot0$$
$$1\cdot8e+6\cdot4n=-20\cdot2$$

$$6\cdot66e+3\cdot24n=+5\cdot4$$
$$6\cdot66e+23\cdot7n=-74\cdot4$$

$$20\cdot46n=-79\cdot8$$

or $n=-3\cdot9$
$e=+2\cdot6$

	E	N
Trial Point =	458980	164390
	+2·6	−3·9
Resected Point =	458982·6	164386·1

These two equations can be solved for values of e and n, which, added algebraically to the co-ordinates of P, give the co-ordinates of T.

It is to be noted that from the points A, B and C a third equation can be obtained, viz. :—

$$e\left(\frac{B_E}{B_T}-\frac{C_E}{C_T}\right)-n\left(\frac{B_N}{B_T}-\frac{C_N}{C_T}\right)=C_{BC}-O_{BC}.$$

But the values of e and n which satisfy the first two (vii) and (viii) must also satisfy this equation, which can be derived from them by subtracting (viii) from (vii).

4. *Comparison of various methods of computation.*

Whatever method is used of working out the observations for a resection, it will always be found that taking the trig points three at a time an exact solution can be found for each three, but that the values obtained from one set of three differ slightly from those obtained from all the others.

This is due to the fact that neither errorless observations nor errorless positions for trig points can be expected.

For each three points a point is found by the computation at which the particular angles recorded in the angle book are subtended by the trig points. If the angles subtended have been observed incorrectly, the positions deduced from the observations will not be the exact position at which the observation was taken.

Since it is impossible to observe angles with absolute correctness, it follows that small differences in the values obtained from each pair of angles must be expected and allowed for.

In computing by absolute methods this can only be done by doing a separate computation for each trio of points and taking the mean of the results.

In semi-graphic methods each three points should give a pin point intersection of three lines on the graph. The final point selected should be the " centre of gravity," of the figure formed by joining the pin points so found.

In the logarithmic method the final position can be found by an extension of the computation as follows :—

Having obtained from pairs of points a series of equations of the form—

$A_1 e + B_1 n + C_1 = 0$ .... (i)
$A_2 e + B_2 n + C_2 = 0$ .... (ii)
$A_3 e + B_3 n + C_3 = 0$ ....(iii)
&c., &c.

Values of e and n which satisfy (i) and (ii) will not satisfy (iii), and mean values of e and n obtained from each pair of equations will exactly satisfy none of them but will satisfy equations of the form—

$A_1 e + B_1 n + C_1 = r_1$ .... (i)
$A_2 e + B_2 n + C_2 = r_2$ .... (ii)
$A_3 e + B_3 n + C_3 = r_3$ ....(iii)
&c., &c.

The mathematically most probable values of e and n are those which make the sum of the squares of the "residuals" r_1, r_2, r_3, &c., a minimum.

The conditions for this are that:—

$$(A_1^2 + A_2^2 + A_3^2 + \ldots)\, e + (A_1 B_1 + A_2 B_2 + A_3 B_3 + \ldots)\, n - (A_1 C_1 + A_2 C_2 + \ldots) = 0.$$

and $(A_1 B_1 + A_2 B_2 + \ldots)\, e + (B_1^2 + B_2^2 + B_3^2 + \ldots)\, n - (B_1 C_1 + B_2 C_2 + \ldots) = 0.$

These are called "normal" equations, and, as can be seen, are obtained by multiplying each of the "observation" equations (i), (ii) and (iii) by its own co-efficients of e and n and adding up each series so obtained.

This is most conveniently done in the actual computation by tabulating these co-efficients as shown in the example given below.

The especial merit of the logarithmic method is the facility with which the mathematically most probable position of the resected point can be found.

Sometimes, however, the discrepancies in the values of the resected point obtained from each three trig points are due not so much to small errors in the observation as to the fact that the positions of one or more of the trig points is erroneous, or that the angle to one of them has been read or recorded wrongly.

It is not uncommon to find for example, that for some reason or other a beacon has been moved from its original position without a note having been made in the triangulation records.

The especial merit of the semi-graphic methods is that an inspection of the graph will generally show at once if any trig point is not in "sympathy" with the others. The cause of the discrepancy having been discovered the observation to that point or points can, if necessary be rejected.

The "tangents" method, although rather confusing to use until it has been thoroughly mastered, is very convenient and quick when all the trig points are already plotted on a single sheet of paper, as is generally the case when the co-ordinates of the trial point are those of a plane table fixing.

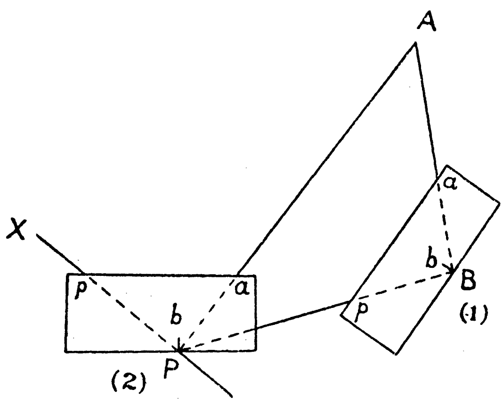


FIG. 39 (*a*).

The " graph " from which the corrections to the co-ordinates of the trial point are determined, can be drawn on the plane table itself using a suitably enhanced scale. The directions of the tangents can be drawn in without the aid of a protractor by means of a small sheet of paper along the edges of which " ticks " can be made to define the required angles.

Thus in Fig. 39A if it is required to draw a line PX making the angle APX = ABP.

A sheet of paper is first placed as in 1 (Fig. 39*a*) and the marks *a*, *b*, *p* made on it, and the paper is laid down as in (2) so that the mark *p* falls on the line PA and the plane table marked at the point *a*.

The tangents method is also easily adapted for marking out on the ground a particular point whose co-ordinates are already known, *e.g.*, for marking the position of a sound ranging microphone. Thus in Fig. 39 (*b*) it is required to mark out the position of a point T.

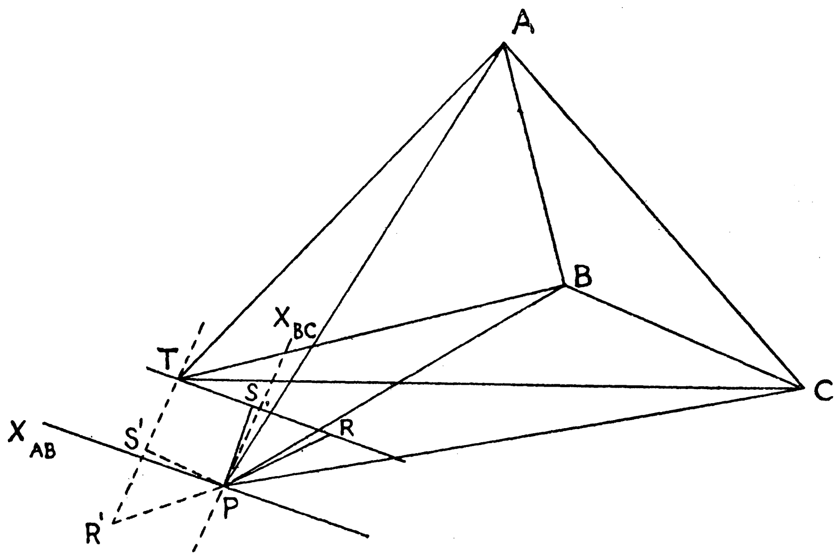


FIG. 39 (*b*.)

The surveyor finds the position as well as he can by plane table or map detail and arrives at P, close to but not exactly at T. He sets up a theodolite or director at P and observes the angle APB, BPC. The angles ATB, BTC should be computed before going out, and the angles ABT, BCT, ACT as well as the lengths AB, BC, AC, AT, BT, CT measured from the map (or computed if necessary).

The points are then taken two at a time, thus :—

The angle ATB is subtracted from ATB, giving a value of say $+ x$ mins.

T therefore lies on a line parallel to PX (the tangent) where APX = ABT very nearly.

The distance of this line from PX is given by—

$$PS = \frac{3}{10} \frac{AT \times BT}{AB} \times X \text{ (AT BT, \&c., being thousands of units).}$$

PS can be laid out on the ground by laying off from A the angle APS = 90° − ABT.

Another point on this line, *e.g.*, R can be found by laying off PR so that SPR = 45° and measuring PR = 1·41 × PS.

In the same way other lines can be quickly laid out on the ground from the points BC and AC.

These lines should intersect nearly in a point, and the theodolite or director can be set up there and a second round of angles taken.

This should agree with the previously computed angles within a minute or so. If necessary, the point T can be laid out again more accurately from this second position.

This is generally quicker than working out a trigonometrical resection at P and laying off the point T by bearing and distance, as the greater part of the logarithmic computation can be done beforehand.

37. Resection from Two Points (an Inaccessible Base).

When three trig points are not available but two can be seen, positions may be resected by observations made at two points.

This is equivalent to triangulation based on an inaccessible base.

Thus if A and B are two trig points (Fig. 40) and C and D stations of observation. The triangles CAD and CBD are first solved using an arbitrary value of CD (*i.e.*, calling CD unity).

This gives the lengths of CA and CB in terms of the same unit, *i.e.*, CD. Then the triangle ACB is solved from the formulæ

$$\frac{A + B}{2} = 90° - \frac{C}{2}$$

$$\tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}$$

Thus obtaining the angles CAB and CBA. The length and bearing of AB is computed from the co-ordinates of A and B and the triangle CAB solved for the true lengths of AC and BC.

From these the length of CD can easily be deduced.

In this way the position of either C or D or both can be found.

From the angles so found, viz., CAB, ABC and the known angle ACB.

The triangle ABC based on the side AB is solved in the ordinary way.

The bearing AC is bearing AB + ∠ CAB.

The bearing BC is bearing BA − ∠ ABC.

The co-ordinates of C are computed from these values, and the computed lengths of AC and BC found from the triangle ABC.

Co-ordinates of D can be computed in a similar manner.

EXAMPLE OF COMPUTATION OF "INACCESSIBLE BASE."

Observations at C.	°	′	″	Observations at D.	°	′	″
To A	0	0	0	To C	0	0	0
„ B	44	12	52	„ A	43	09	33
„ D	88	47	39	„ B	97	44	28

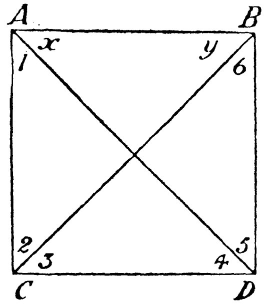


Fig. 40.

Triangle ACD.	°	′	″	Triangle BCD.	°	′	″
C =	88	47	39	C =	44	34	47
D =	43	09	33	D =	97	44	28
sum =	131	57	12	sum =	142	19	15

	°	′	″			°	′	″	
log sin	43	09	33	= $\bar{1}$·8350735	log sin	97	44	28	= $\bar{1}$·9960240
log sin	131	57	12	= $\bar{1}$·8713917	log sin	142	19	15	= $\bar{1}$·7862113
				$\bar{1}$·9636818					0·2098127

b = ·9198

$a - b$ = ·7013

$a + b$ = 2·5409

a = 1·6211

C/2 = 22° 06′ 26″

log $a - b$ = $\bar{1}$·8459038
log $a + b$ = 0·4049876

$\bar{1}$·4409162
log cot C/2 = 0·3912550

$\bar{1}$·8321712

$$\frac{A + B}{2} = 67°\ 53'\ 34''$$

$$\frac{A - B}{2} = 34°\ 11'\ 42''$$

A = 102° 05′ 16″
B = 33° 41′ 52″

CHAPTER XI.

FIELD ASTRONOMY.

38. DEFINITIONS.

1. By observation of the heavenly bodies, *i.e.*, the sun, moon, the planets, or the stars, it is possible to obtain absolute determinations at any place of—

(*a*) The local time.
(*b*) The azimuth, or direction of true north.
(*c*) The latitude.
(*d*) The longitude.

2. With the instruments which can be used by the artillery surveyor in the field it is not possible to determine position, as defined by latitude and longitude, with sufficient accuracy to be of any use for

gunnery. The observations required for this purpose are not considered in detail in this book.

Determination of time and azimuth can, however, be made with more than sufficient accuracy for artillery requirements, and have an immediate practical value.

3. For astronomical observations in the field only the sun or stars are used, so that the relative movements of the earth with relation to these only need be considered.

Both the sun and stars have proper motions in relation to each other, but they are situated at so great a distance from the earth that for practical purposes these motions may be neglected. Their *apparent* movements will, therefore, depend only on the actual movements of the earth with reference to any one of them.

4. The earth has two proper motions in the solar system.

(*a*) A motion round the sun in a fixed orbit.
(*b*) A revolution round its own axis.

The path of the earth round the sun is an ellipse of which the sun is one focus. This path lies entirely in a single plane called the *ecliptic.* Though elliptical, its eccentricity is small, and it is approximately a circle whose mean radius is about 93,000,000 miles. The time taken by the earth to make one complete revolution round the sun is something over 365 days, and constitutes the *solar year.*

The complete revolution of the earth round its own axis constitutes the solar day. The plane of this revolution does not coincide with the plane of its revolution round the sun. That is to say, the axis of its own proper revolution is not parallel to the axis of its revolution round the sun. It is inclined to the plane of the ecliptic at an angle of about $66\frac{1}{2}$ degrees.

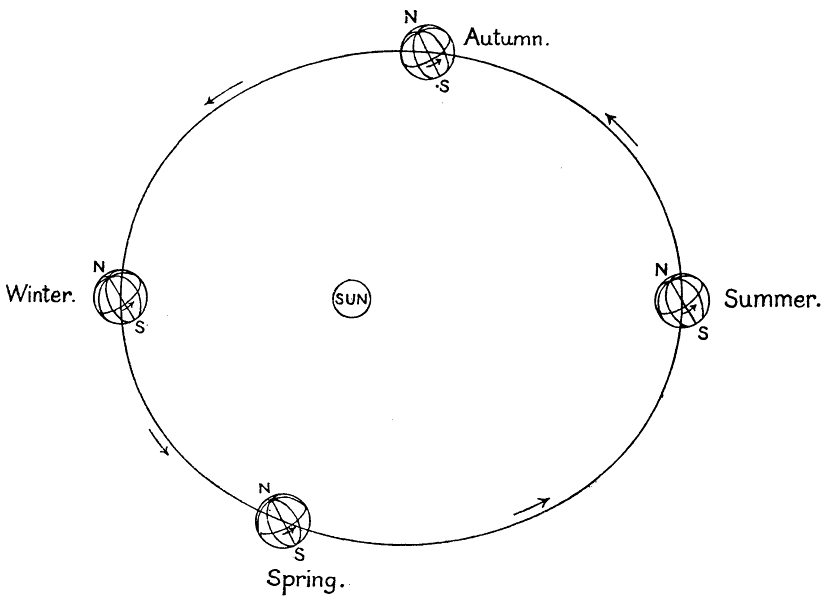


FIG. 41.

Fig. 41 shows the relative positions of sun and earth at different times of the year. Imagining the observer situated above the plane of the ecliptic and looking down on it so that the North Pole of the earth's axis is towards him, the direction of the two motions are as shown by the arrows.

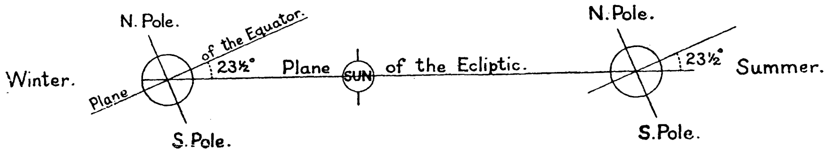


FIG. 42.

Fig. 42 shows the relative positions of sun and earth as viewed by an observer situated *in the plane of the ecliptic at the position marked "Spring" in Fig.* 41.

5. The combined effect of these two motions of the earth is that the sun, stars and other heavenly bodies *appear* to revolve round the earth, rising in the east and setting in the west, but altering their apparent positions in the sky slightly each day. That is to say, the time at which the sun or a star rises, alters slightly from day to day, and its distance from the horizon at the highest point of its path also changes.

6. For purposes of observation, it is convenient to regard the heavenly bodies as being situated on an imaginary sphere of which the earth is the centre, the position of each body being taken as the point, where a line joining it to the centre of the earth cuts this sphere.

This "celestial sphere" will appear to revolve round the earth about an axis which is coincident with and a prolongation of the earth's own axis.

7. Since the position of the earth's centre does not alter as the earth revolves the positions of the heavenly bodies on the *celestial sphere* will be fixed* even though these positions with reference to an observer on the earth's surface *appear* to change when referred to terrestial objects. It is from measurements of these apparent changes that information required for the determinations referred to in para. 1 are obtained.

8. In astronomical work the following are the principal definitions etc., applicable to the celestial sphere :—

Great Circle—is a circle on the surface of the sphere, whose plane contains the centre of the sphere.

Small Circle—is a circle on the surface of the sphere whose plane does not pass through the centre of the sphere.

Celestial Equator—is the intersection of the plane of the earth's equator with the celestial sphere.

Celestial Poles—are the points where the earth's axis produced cuts the celestial sphere.

*** Actually the stars have certain apparent proper motions which are a combination of small movements known as Aberration, Precession, Mutation and Proper motion, details of which will not be described here (*see also* para. 10).**

Zenith.—The point on the celestial sphere vertically over the observer at any point.

Horizon.—The intersection of a plane, passing through the observer at 90° from the zenith, with the celestial sphere.

Elevated Pole.—That celestial pole which is above the observer's horizon. To an observer in the Northern Hemisphere this will be the North celestial pole.

Vertical Circle.—Any great circle on the celestial sphere which passes through the zenith.

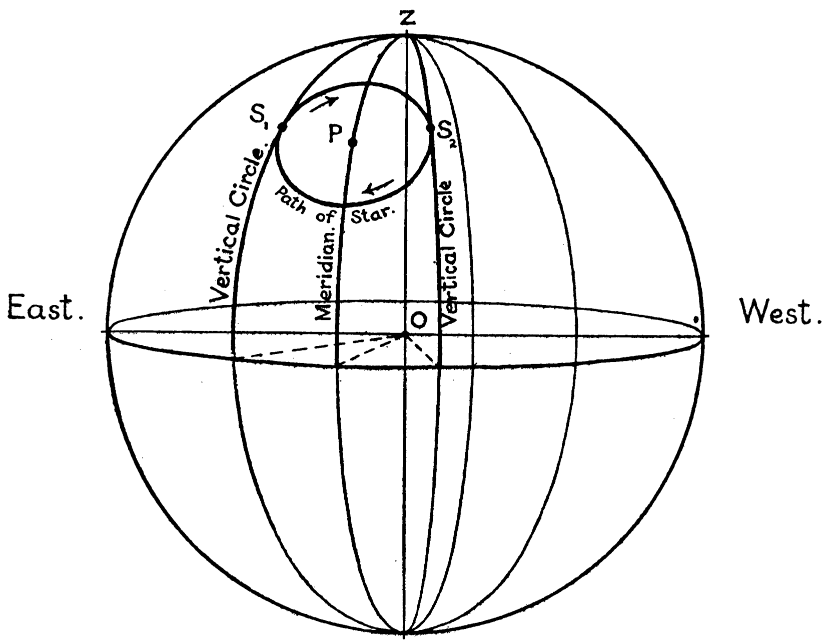
Meridian.—A vertical circle passing through the two poles.

Prime Vertical.—A vertical circle which cuts the meridian at right angles.

Transit.—A heavenly body is said to "transit," when it crosses the observer's meridian. That transit which occurs on the same side of the elevated pole as the observer's zenith is called "upper" transit. That which occurs on the opposite side of the elevated pole is called the "lower" transit.

Circumpolar star.—When both transits of a star take place above the observer's horizon, the star is known as a "circumpolar" star.

Elongation.—A star is said to be at elongation when the vertical circle through it touches, but does not cut its path in the



Circumpolar Star at Elongation.
Position of the observer imagined to be outside and North of the Celestial Sphere.

Fig. 43.

heavens. Thus in Fig. 43 if O is the observer, Z the zenith, P the elevated pole, S_1 and S_2 represent the positions of a star at elongation. In the Northern Hemisphere P would be the North Pole, S_1 would be called east elongation and S_2 west elongation

9. The positions of the heavenly bodies on the celestial sphere are defined by co-ordinates analogous to the spherical or geographical co-ordinates used for defining the terrestial points.

That co-ordinate, which corresponds to the *latitude*, is known as the *declination*, which is the angular distance between the star and the plane of the equator.

Declination is usually indicated by the Greek letter δ and is given in degrees, minutes, and seconds North or South as the case may be.

The great circle passing through the star and cutting the equator at right angles is called the *declination circle* of the star.

That co-ordinate, which corresponds to the *longitude*, is known as the *right ascension*, which is the angular distance between a great circle passing through the star and the poles, and another great circle passing through the poles and an arbitrary point known as the *first point of Aries*.

The right ascension is usually denoted by the letters R.A., and is expressed in hours, minutes, and seconds of *time*, 24 hours corresponding to 360 degrees.

10. The astronomical co-ordinates of the heavenly bodies on every day in the year are published annually in the *Nautical Almanac* prepared at Greenwich Observatory.

There are two editions of this publication. In the complete edition the co-ordinates of the stars will be found under " Apparent Places of Stars."

In the abridged edition these co-ordinates are given simply under the heading " Stars."

In both publications the co-ordinates of stars are given in the order of their right ascensions.

It will be noted that both declination and right ascension of a star change slightly during the year. These changes are due to the certain small movement, referred to in the note to Para. 8.

In effect, since these motions are known, the precise position of a star at any date can be predicted, and the changes being slow, they have no effect during the time an observation is in progress. The stars may therefore be regarded as fixed points whose positions are those given in the *Nautical Almanac* for the particular date on which the observations take place.

11. *The First Point of Aries*, which is the zero from which right Ascensions are measured (and which corresponds to the meridian at Greenwich in measures of longitude), is the point on the celestial sphere where it is cut by the line of intersection of the plane of the equator with the plane of the ecliptic, when this line passes through the centre of the sun, at the Vernal Equinox.

Thus in Fig. 44 the position of the earth is shown at two points in its orbit. It can be seen that the planes of the equator and the ecliptic

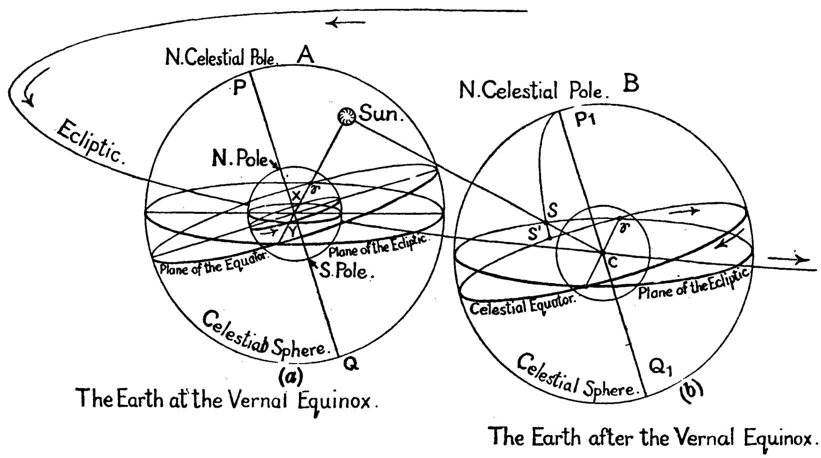


FIG. 44.

intersect in the line XY, and that as the earth moves round the sun this line remains parallel to itself. At the Vernal and Autumnal Equinoxes this line produced passes through the sun and will cut the imaginary celestial sphere, which revolves round the line PQ or P_1Q_1, at some point. This point is denoted by the Greek letter γ and is called the "First Point of Aries," and the great circle passing through it and the poles is the zero from which right ascensions are measured.

39. Solar and Sidereal Times.

1. A solar day is the interval between two successive upper transits of the sun.

Kepler's second law of motion states that "As a planet moves round the sun its radius vector passes over equal areas in equal times."

Fig. 45 shows the position of the radius vector (*i.e.*, a line joining the centres of the earth and sun) in different positions of the earth's orbit.

From Kepler's law it follows that, if the time taken by the earth to travel from E_1 to E_2, from E_2 to E_3, E_3 to E_4 is the same, the areas $E_1\,SE_2$, $E_2\,SE_3$, $E_3\,SE_4$, will be equal.

Since, however, the orbit of the earth is an ellipse, the distances from the sun, *i.e.*, SE_1, SE_2, &c., will not be equal, and consequently, the *lengths* $E_1\,E_2$, $E_2\,E_3$, &c., traversed in a given time will also vary according to the position of the earth in its orbit.

2. The effect of this variation in velocity round the sun is that the interval between successive transits over any particular meridian is not uniform, but varies from day to day throughout the year.

The solar day, or any fraction of it, is not therefore a practicable unit for measurement of time, since it is not possible to construct clocks which will vary their rate according to the position of the earth in its orbit.

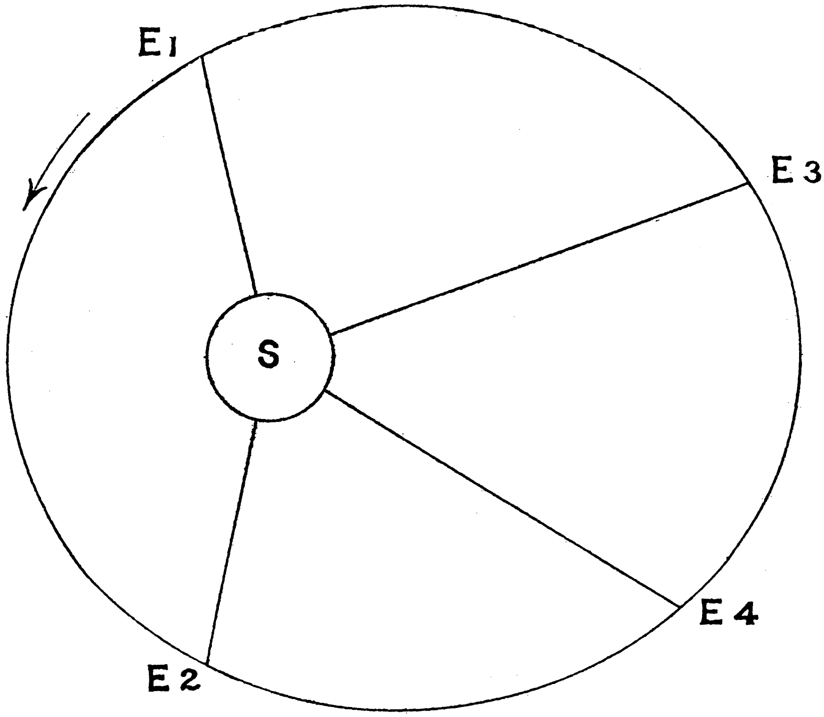


Fig. 45.

A mean and constant value of the solar day is therefore taken as the unit and subdivided into hours, minutes and seconds. A *mean* solar day is considered as the interval between two successive upper transits of a fictitious "Mean Sun." The position of this mean sun can be calculated. The difference in R.A. between the mean and real sun is called the "equation of time."

3. This equation varies throughout the year and is given in the Nautical Almanac on page 1 of every month.

By means of this equation the position of the mean sun, which has no real existence, can be deduced from observations made to the real sun.

The local time at any point is the interval which has elapsed since the sun was on the meridian at that point.

Local Apparent Time (L.A.T.) is the interval from the transit of the real sun. Local Mean Time (L.M.T.) is the interval from the transit of the fictitious *mean sun.*

4. It should be noted that at present the astronomical day commences at noon, while the civil day commences at midnight, and that time intervals are commonly measured with clocks or watches made to record as nearly as possible units of *mean time.*

From 1925 onwards the Nautical Almanac will be made out for an astronomical day identical with the civil day.

For example, at present:—

5 a.m. on Aug. 9th, civil date, corresponds to 17 hours Aug. 8th (Astronomical).

6 p.m. on Aug. 9th, civil date, corresponds to 06 hours Aug. 9th (Astronomical).

5. A solar year is the time taken by the earth to travel once round its orbit, commencing and ending with its position at the Vernal Equinox. During this period it completes 365·2422 revolutions round its own axis.

6. A sidereal day is the interval between two successive transits of the First Point of Aries. In Sect. 40, para. 11, it has been explained that the First Point of Aries is an imaginary point on the "*celestial sphere*," which, in its turn, is an imaginary sphere whose *apparent* movements depend solely on the *actual* rotation of the earth on its own axis.

This rotation proceeds at a uniform rate. The length of the sidereal day does not therefore depend in any way on the earth's position with reference to the sun, and is therefore of constant length.

The sidereal day is, like the solar day, divided into hours, minutes and seconds of *sidereal* time.

7. *Relation between Sidereal and Solar times.*—At the Vernal Equinox, the commencement of the astronomical year, the position of the sun coincides with that of the First Point of Aries on the celestial sphere.

Fig. 44 shows the earth at the Vernal Equinox and also shows its position after it has travelled a certain distance in its orbit.

At (*a*) the arrow shows the *actual* direction of the earth's revolution, at (*b*) the arrow indicates the *apparent* direction of the revolution of the celestial sphere.

It can be seen that, when at (*a*), the sun's position on the celestial sphere coincides with that of γ, while at (*b*) it is at S somewhat behind it. The First Point of Aries will, therefore, transit any particular meridian before the sun; the time interval between the two transits being the time taken by the earth to revolve through the angle of $SC\gamma$.

It can be seen that, as the earth moves further and further from the Vernal Equinox, this interval increases until as the earth completes its orbit it amounts to 360° or 24 hours. In other words, the First Point of Aries in the year transits once more than the sun. The solar year of 365·2422 mean solar days represents 366·2422 sidereal days.

This fact gives the relation between Mean and Sidereal time.

The sidereal day of 24 *sidereal hours* is somewhat shorter than the solar day of 24 *mean time hours.*

1 mean time hour = 1 hour + 9·86 seconds (sidereal hours and seconds).

1 sidereal time hour = 1 hour — 9·83 seconds (mean time units).

From these relations, which are constant, an interval recorded in mean time units can be expressed in terms of sidereal time or *vice versa.*

8. The following abbreviations are commonly used, for denoting times in astronomical work.—

G.M.T. = Greenwich Mean Time. L.M.T. = Local Mean Time.

G.A.T. = Greenwich Apparent Time.	L.A.T. = Local Apparent Time.
G.S.T. = Greenwich Sidereal Time.	L.S.T. = Local Sidereal Time.
G.M.N. = Greenwich Mean Noon.	L.M.N. = Local Mean Noon.
G.A.N. = Greenwich Apparent Noon.	L.A.N. = Local Apparent Noon &c., &c.

9. *Examples.*

(1) At Greenwich on January 1st, 1921, G.M.T. is 3 p.m. What is the G.S.T. at this moment?

From the Nautical Almanac for 1921, page 1, January, it is found that G.S.T. of G.M.N. = 18 hours 42 minutes 22·6 seconds.

That is to say the Mean Sun transits at Greenwich 18 hours 42 minutes 22·6 seconds *sidereal units* after γ.

3 p.m. represents an interval of 3 M.T. hours after the transit of the Mean Sun at Greenwich.

3 M.T. hours = 3 hrs. + 3 × 9·86 secs. sidereal units.
= 3 hrs. 0 mins. 29·58 secs. sidereal.

The interval from sidereal noon, which is the sidereal time, is therefore—
18 hrs. 42 mins. 22·6 secs. + 3 hrs. 0 mins. 29·58 secs.
or, G.S.T. = 21 hrs. 42 mins. 52·18 secs.

(2) In longitude 30° W., on January 1st, 1921, the L.M.T. is 5 hours. What is the L.S.T.?

The mean sun travels 360° in 24 hrs.
∴ The mean sun travels 30° in 2 hrs.

During these two hours γ gains 9·86 secs. S.T. per hour or 19·72 secs. (S.T.).

L.S.T. of L.M.N. = (G.S.T. of G.M.N.) + 19·72 secs.

This is the *sidereal interval* between the transit of γ and the mean sun across the meridian at 30° W.

L.M.T. 5 hrs. is 5 hrs. M.T. later and is therefore 5 hrs. + 5 × 9·86 secs. S.T. later.

The L.S.T. is therefore :—

	h.	m.	secs.
G.S.T. of G.M.N.	18	42	22·6
	+ 0	0	19·72
	+ 5	0	49·30
	23	43	31·6

(3) In longitude 42° 39′ 30″ E. the L.S.T. on May 20, 1921, is 5 hrs. 46 mins. 16·6 secs. What is the L.M.T.?

Convert 42° 39′ 30″ to hours at 15° per hour = 2·84 hours. From the N.A. G.S.T. of G.M.N. = 3 hrs. 50 mins. 23·6 secs, and, as the longitude is *East*, L.S.T. of L.M.N. will be *less* than this by 2·84 × 9·86 seconds : *i.e.*, 27 secs.

	h.	m.	secs.
... L.S.T. of L.M.N. is	3	50	23·6
	—		27·0
	3	49	56·6 (S.T.)

The epoch we are considering is 5 hrs. 46 mins. 16·6 secs. (L.S.T.).

Therefore the sidereal interval since L.M.N. is the difference between these two sidereal times: viz. :—

Sidereal Interval = 1 hr. 56 mins. 20·0 secs.

To convert this to M.T. we have to subtract:—

	secs.
For 1 hour	9·83
For 56 mins.	9·17
For 20 secs.	0·06
	19·06

Sidereal Interval = 1 hr. 56 min. 20·00 secs.
L.M.T. = 1 hr. 56 mins. 0·94 secs.

10. It is to be noted that, in converting longitude to hours, it is immaterial whether M.T. or S.T. hours are referred to.

Although the length of M.T. and S.T. hours are different the angular interval of longtitude corresponding to an hour is the same.

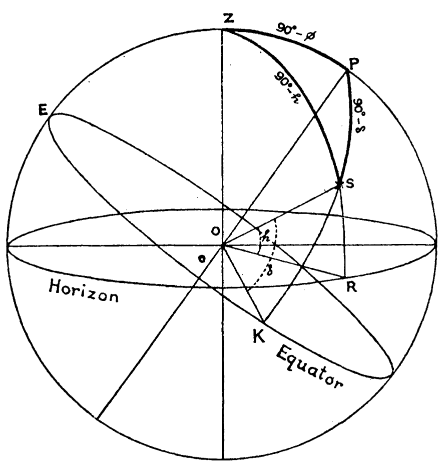
The apparent movement of the sun is slower than that of γ, but the M.T. hour is correspondingly longer.

Expressed in other words, the Mean Sun moves through an angular distance of 360° in 24 M.T. hours, and γ moves through the same angular distance in 24 sidereal hours.

The relation between hours and angles is therefore the same whichever unit is used.

40. The Astronomical Triangle.

1. A heavenly body at any moment forms with the zenith and the elevated pole a spherical triangle known as the "Astronomical Triangle." This triangle is shown in Fig. 46, lettered ZSP, where Z is the zenith, S the star or heavenly body, and P the elevated pole.



The Astronomical Triangle.

Fig. 46.

The elements of this triangle are :—

The side ZP subtending the angle ZOP = 90° — ZOE = 90° — Observers *latitude* (written 90° — φ).

The side ZS subtending the angle ZOS = 90° — SOR = 90° — star's altitude (written 90° — h).

The side PS subtending the angle POS = 90° — SOK = 90° — star's declination (90° — δ).

The angle ZPS is the angle between the Meridian and the *declination* circle of the star.

It is called the " Hour angle " and denoted by the letter " t."

The angle SZP is the angle between the Meridian and a *vertical* circle through the star.

It is the *Azimuth* of the star and is denoted by the letter " A."

The angle ZSP is the angle between the declination circle and the vertical circle and is known as the parallactic angle. When the star is at elongation the parallactic angle is 90°. Except in this particular case the parallactic angle does not enter into the computations required in Field Astronomy.

2. Of these six elements, if any three are known, the triangle can be solved to give the value of the remaining three. One of them, namely the declination, can always be obtained from the Nautical Almanac. Another, the altitude, can always be observed, since a spirit level or plumb-line can be used to indicate the position of the zenith.

The third element which must be known may be either the latitude or the longtitude, since from the latter the local time (L.M.T. of G.M.N.) can be obtained, and from it the hour angle.

3. Two particular cases of this triangle should be noted, one, already referred to, when the parallactic angle is a right angle and the other when the star is on the Meridian when the Azimuth angle is zero.

41. Astronomical Observations.

1. In practice the observations required for Field Astronomy resolve themselves into one or more of three measurements according to the data available with regard to the observer's position.

(*a*) Measurement of the vertical angle or *altitude* of the star.

(*b*) Measurement of the horizontal angle between the star and some " reference object," as a fixed reference line for *azimuth*.

(*c*) Measurement of the time at which observation has been made ; that is measurement of the interval which has elapsed between local mean noon or other fixed epoch and the moment of the observation.

2. In dealing with these measurements the observer is regarded as being at the centre of the celestial sphere. The distance of the stars from the earth is so great that for practical purposes the earth's dimensions may be neglected and the whole earth treated as a single point. In observing the sun, however, the dimensions of the earth cannot be neglected and must be allowed for (*see* para. 5).

Before they can be used for calculation the field observations have to be corrected for refraction, and if necessary, for clock or watch rate.

3. *Measurement of the altitude of a star* may be made either with a sextant, using a "mercury horizon," or with a theodolite.

In the first case the angle between the star and its reflection in a bath of mercury is measured; this angle is twice the altitude.

In the second case, the angle between the star and the horizon as indicated by a spirit level is measured. In this case it is not possible to ensure that the method of attaching the spirit level to the base plate of the theodolite will always be such as to enable the latter to be levelled exactly.

Errors due to dislevelment can be overcome by changing "face," but since the star will move while the operation of changing face is being performed, this change of face will only eliminate the error if the movement of the star is uniform, *i.e.*, when the mean of the times of observation exactly corresponds to the mean of the elevations observed on the two faces.

This will obviously only be the case when the star is on the prime vertical. The further a star is from the P.V. the greater will be the variation in rate of change of elevation in equal successive periods of time.

For this reason, when the rate of change of altitude is considerable, the star selected for observation should be as nearly as possible on the P.V., *i.e.*, either due east or due west of the observer. The observations are generally taken F.R., F.L., F.L., F.R., and the time of each noted. The mean of the four observed altitudes is then considered to be that of the star at the mean of the four recorded times and, after correction for refraction and watch error, is used for computation.

4. *Refraction.*—When a ray of light traverses two transparent media of different densities the ray is bent at their plane of junction; the amount of this bending depends on the angle the ray makes with this plane, being zero when the ray is normal to the plane, and increasing as the angle the ray makes with the normal increases.

Owing to the fact that the density of the atmosphere diminishes as the distance from the earth increases, a ray of light traversing the atmosphere has to pass through media of different densities. The amount of the refraction of a terrestrial ray will therefore depend on its length and the heights of the points at each end of it.

The rays used in astronomical work, however, traverse the whole atmosphere, and the amount of refraction depends solely on the angle at which the ray strikes it. That is to say on the altitude of the point from which it emanates.

The amount of this refraction varies with the place, season and time of day, but for angles of elevation exceeding 10° the laws governing it are fairly well known. Provided the observed altitudes are not less than 10° the correction to be made for refraction can be made with considerable precision. These corrections may be taken from tables given in Chambers Logarithm Tables, the Text Book of Topographical Surveying, and in Part II of this Manual.

These tables are not rigorously accurate, since the refraction depends to some extent on the state of the atmosphere, but it is often possible to eliminate possible errors by taking two observations in such a way that any inaccuracies there may be affect the result in opposite ways, and then taking the mean of the two.

5. *Observations to the sun.*—Two special factors affect observations to the sun :—

Parallax in altitude.
Semi-diameter.

(*a*) *Parallax in altitude.*—The length of the sides of the spherical triangle are measured at the centre of the celestial sphere.

Observations made to a *star* from any point on the earth's surface may be considered to be made from this centre. But, owing to the comparative proximity of the sun to the earth, this assumption cannot be made in the case of solar observations.

In the case of a star (Fig. 47 (*a*)) the observed altitude is equal to the geocentric altitude.

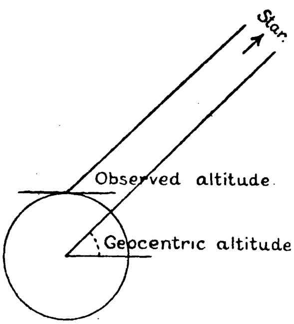


FIG. 47 (*a*).

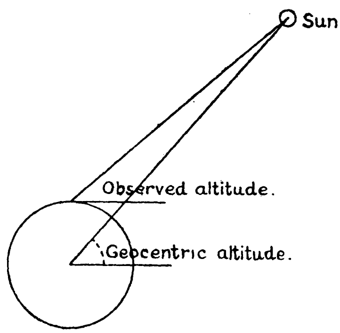


FIG. 47 (*b*).

In the case of the sun (Fig. 47 (*b*)) the geocentric altitude will be greater than the observed altitude by the small angle subtended at the sun by the radius of the earth at the point of observation. This small angle is known as the sun's parallax in altitude: it is greatest when the sun is on the observer's horizon, and is zero when the sun is at its zenith.

Tables of parallaxes are given in the " Textbook of Topographical Surveying," page 368, and in Part II of this Manual.

(*b*) *Semi-diameter.*—The positions given for the sun in the *Nautical Almanac* are those of its centre. It is not possible accurately to bisect the disc of the sun with the cross-hairs of the theodolite. Consequently, in taking successive faces, the cross-hairs are brought tangential to the sun's disc on opposite limbs, as in Fig. 48.

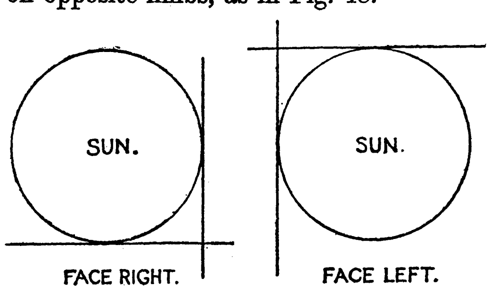


FIG. 48.

The altitudes and azimuths will be read too large on one face, and too small on the other: the mean of the faces will give the readings to the sun's centre.

A correction for semi-diameter is only necessary when it is impossible to make the observations in this manner.

42. Observations for Time.

1. A knowledge of time may be required as a preliminary to subsequent observations for azimuth or latitude or for its own sake.

It may be determined in one or other of the following ways :—

(I) By single altitudes on or near the P.V.

(II) By "equal altitudes."

Method I.—The observation of the star is effected as described in Section 43.

The observed altitude, corrected for refraction, etc., gives the side ZS of the astronomical triangle which is then solved from the formula

$$\tan\frac{t}{2} = \sqrt{\cos s \sin (s - h) \operatorname{cosec} (s - \phi) \sec (s - p)}$$

where

t = hour angle.
h = observed altitude.
ϕ = latitude.
p = polar distance (90 — declination).
$s = \dfrac{h + \phi + p}{2}$

The R.A. ($\pm\, t$ according as the star is $\dfrac{\text{West}}{\text{East}}$ of the Meridian) gives the L.S.T. of observation, from which the L.M.T. of the observation can be deduced.

Method II.—If a star can be observed at the same altitude both before and after it crosses the meridian, it is evident that, since its declination is constant, it will have transited at the mean of the observed times. A comparison of this time of transit with the R.A. of the star will give the error of the time-keeper.

When the sun is used with this method, the declination cannot be regarded as constant, and the mean of the observed times must be corrected.

This correction is—

$$\frac{T \times d\,\delta}{15}(\tan\phi \operatorname{cosec} T - \tan\delta \cot T)$$

where

T = half the interval between the two observations.
$d\delta$ = the change of declination of 1 hour.
δ = the declination of the sun when on the meridian.
ϕ = the latitude.

The correction is + when the sun is moving away from the elevated pole and — when approaching it.

In using this method it is necessary to take more than one reading on each side of the meridian. To do this the instrument is set forward to readings of altitude differing by say 20′ on the upward journey, noting the times at which the star passes the cross-hair, and setting the instrument at the same readings for the downward journey.

Each pair of readings gives a determination of time, and the mean of all gives the final result.

Example of method I:—

LOCAL MEAN TIME FROM ALTITUDE OF A STAR.

Place ... *Larkhill.* Date ... 28 Nov., 1910. Star ... *a Aquilae.*
Latitude (ϕ) ... 51° 12′ 01″ Longitude (in time) ... 00h. 07m. 08s.
Barometer ... 28·90″. Thermometer *33°.* Level value = *20″.*

Observed Times.	°	′	″	*E*	*O*	Observed Altitude.	°	′	″	Mean Alt. °	′	″
FL	*7*	*21*	*49*	*11·5*	*9·5*	*C*	*25*	*23*	*20*	*25*	*25*	*03*
						D	*25*	*26*	*46*			
FR	*7*	*23*	*22*	*10·0*	*10·5*	*C*	*25*	*13*	*50*	*25*	*11*	*55*
						D	*25*	*10*	*00*			
FR	*7*	*25*	*07*	*10·0*	*10·5*	*C*	*24*	*57*	*46*	*24*	*55*	*50*
						D	*24*	*53*	*54*			
FL	*7*	*27*	*08*	*11·5*	*9·5*	*C*	*24*	*36*	*00*	*24*	*37*	*56·5*
						D	*24*	*39*	*53*			
FL						*C*						
						D						
FR						*C*						
						D						
4)	*29*	*37*	*26*	*43·0*	*40·0*					4)*100*	*10*	*44·5*
Mean =	*7*	*24*	*21·5*	*40·0*		Mean Alt.				*25*	*02*	*41·125*
				8) *3·0*		Level corr. ± —						*7·5*
				·375 × 20						*25*	*02*	*33·625*
						Refraction —					*2*	*4·25*
Level Corr. ±				— *7·5″*								
						True Alt. (h)				*25*	*00*	*29·38*

Refraction for *25° 2·75′* = *2′ 4·75″.*
Corr for Bar ± *—1·10″* = *— 5.*
Corr. for Ther. ± = *+ 4·5.*

Refraction = *2′ 4·25″.*

	°	′	″
Stars RA	*19*	*46*	*24·5*
Stars Declination ...	*8*	*37*	*54·74*
	90	00	00
Stars Polar distance (p)	*81*	*22*	*05·26*

		°	′	″		
True Alt. (h)	=	*25*	*00*	*29·38*		
Latitude (ϕ)	=	*51*	*12*	*01*		
Polar Distance (p)	=	*81*	*22*	*05·26*		
	2)	*157*	*34*	*35·64*		
S	=	*78*	*47*	*17·82*	log cos	= *9·2887724*
$S-h$	=	*53*	*46*	*48·44*	log sin	= *9·9067412*
$S-\phi$	=	*27*	*35*	*16·82*	log cosec	=*10·3343146*
$S-p$	=	*2*	*34*	*47·44*	log sec	=*10·0004404*

Divide sum by 2)*39·5302686*

$$\log \tan \frac{t}{2} = 9{\cdot}7651343$$

$$\therefore \frac{t}{2} = 30^\circ\ 12'\ 41{\cdot}5''$$

LOCAL MEAN TIME FROM ALTITUDE OF A STAR—*cont.*

	°	′	″
G.S.T. of G.M.N. =	*16*	*26*	*1·56*
Corr. for Long. in time ... =+			*1·17*
W + E −			
∴ L.S.T. of L.M.N.=	*16*	*26*	*2·73*

∴ t = *60° 25′ 23″*

	h.	m.	s.
∴ t in time =	*4*	*01*	*41·6*
Stars R.A. =	*19*	*46*	*24·5*
∴ L.S.T. of observ. =	*23*	*48*	*06·1*
but L.S.T. of L.M.N. =	*16*	*26*	*02·73*
∴ Sid. int. of obsn. from L.M.N.=	*7*	*22*	*03·37*
But 7 Sid. hrs. = 7 mean hrs. −		*1*	*8·81*
22 Sid. mins. = 22 mean mins. −			*3·60*
3 Sid. secs. = 3 mean secs. −			*·01*
		1	*12·42*
∴ L.M.T. at instant of observation =	*7*	*20*	*50·95*
But mean of observed times=	*7*	*24*	*21·5*
∴ Watch is fast of L.M.T.		*3*	*30·5*

TIME BY ALTITUDES OF THE SUN.

Place ... *Larkhill.* Date ... *6 July, 1922.*
Latitude (ϕ) ... *51° 12′ 01″.* N. Longitude (in time) ... *0*h. *7*m. *8*s. *W.*
Barometer *30·6 In.* Thermometer *61° F.* Level value = *20″.*

	Observed Times.			*E*	*O*		Altitudes.			Mean Alt.		
	°	′	″				°	′	″	°	′	″
FL	*10*	*11*	*45*	*9·5*	*11·5*	*C*	*45*	*18*	*17*	*45*	*18*	*32*
						D	*45*	*18*	*47*			
FR	*10*	*14*	*01*	*9·5*	*11·5*	*C*	*45*	*34*	*08*	*45*	*34*	*24*
						D	*45*	*34*	*40*			
FR	*10*	*15*	*32*	*9·5*	*11·5*	*C*	*45*	*43*	*32*	*45*	*43*	*51*
						D	*45*	*44*	*10*			
FL	*10*	*17*	*49*	*9·5*	*11·5*	*C*	*45*	*58*	*07*	*45*	*58*	*17*
						D	*45*	*58*	*27*			
FL						*C*						
						D						
FR						*C*						
						D						
4)	*40*	*59*	*07*	*38*	*46*				4)	*182*	*35*	*04*
Mean =	*10*	*14*	*47*		*38*				Mean Alt. =	*45*	*38*	*46*
				8) *8*					Level corr. ± =			*+20*
				1								
					×*20*				Corrected observed Alt.=	*45*	*39*	*06*

Level corr. ± = *+20″*

TIME BY ALTITUDES OF THE SUN—*cont.*

(1)		°	′	″
Refraction for 45° 39′ ...=	−			*57·08*
Corr. for Bar. =	± +			*1·1*
Corr. for Ther. =	± −			*1·3*
Corr. for Parallax ...=	+ +			*6·0*
Total corr. to Alt. ...=	± −			*51·28*
Corrected obsd. Alt. ...=		*45*	*39*	*06*
True Alt. (*h*) =		*45*	*38*	*14·7*

(2)		h.	m.	s.
Mean of obsd. Times ...=		*10*	*14*	*47*
Longitude (*W*+ *E*−) =	± +		*7*	*8*
Approx G.M.T. of observation =		*10*	*21*	*55*
Approx. interval from nearest G.M.N. =		*1*	*38*	*05*
Equation of time at nearest G.M.N....=			*4*	*34·3*
Int. from G.M.N. × hourly variation ...=	± −			*·666*
Equation of time at time of observatn. =	± +		*4*	*33·6*

(3)		°	′	″
Declination of Sun at nearest G.M.N....=		*22*	*39*	*2·1*
Int. from G.M.N. × hourly variation ...=	+			*24·1*
Declination at time of observation (N− S+) ...=	± −	*22*	*39*	*26·2*
Declination of elevated Pole (N+ S−) ...=	± +	90	00	00
Polar Distance (*p*) ...=		*67*	*20*	*33·8*

h=*45 38 14·7*
φ=*51 12 01*
p=*67 20 33·8*
2)*164 10 49·5*

S=*82 05 25*	log cos	=*9·1386583*
S-*h*=*36 27 10*	log sin	= *9·7739035*
S-*φ*=*30 53 24*	log cosec	=*10·2895513*
S-*p*=*14 44 51*	log sec	=*10·0145479*

Divide sum by 2)*19·2166610*

log tan $\frac{t}{2}$ = *9·6083305*

$\frac{t}{2}$ = *22° 05′ 18″*

t = *44 10 36*

	h.	m.	s.
∴ *t* in time =	*2*	*56*	*42·5*
If before noon subtract from	*24*	*0*	*0*
L.A.T. at time of obsn. =	*21*	*03*	*17·5*
but Equation of Time = ± +		*4*	*33·6*
∴ L.M.T. at time of obsn. =	*21*	*07*	*51*
Mean of obsd. times =	*22*	*14*	*47*
∴ Watch is fast ... =	*1*	*06*	*56*

43. OBSERVATION OF AZIMUTH.

1. Astronomical observations of azimuth may be required for correctly orienting a system of triangulation, for control of direction in traverse work, as a check on trigonometrical interpolation, and for laying out a line of fire in an enclosed position, as well as for other purposes.

There are four methods which may be used, viz.—

(i) By altitudes on the P.V.
(ii) By hour angles on the P.V.

(iii) By circumpolar star at elongation.
(iv) By Polaris or other close circumpolar star at any time.

In observing for azimuth it is necessary to select a reference object at a suitable distance from the observer's position. For night work, *i.e.*, when stars are used for the observation, this may be a picket about 300 yards from the observing station, marked with a lamp.

2. *Method I.*—In this method it is necessary to intersect the star with the cross-hair, or in the case of the sun, to bring both horizontal and vertical cross-hairs simultaneously into contact with the edge of the sun's disc.

The procedure is similar to that given in Section 43, except that it is unnecessary to record the exact time of observation but both horizontal and vertical angles should be read. Thus—

Face left	Horizontal angle to R.O.
Face left	Vertical and Horizontal angles to star.
Face right	„ „ „ „
Face right	Horizontal angle to R.O.
Face right	Vertical and horizontal angles to star.
Face left	„ „ „ „
Face left	Horizontal angle to R.O.

The azimuth of the star is computed from the formula

$$\tan \frac{A}{2} = \sqrt{\sec s \sin (s - h) \sin (s - \phi) \sec (s - p)}$$

using the same notation as before.

EXAMPLE—

AZIMUTH.

BY SUN OR STAR WITH THE THEODOLITE.

Sun or Star ... *a Aquilae.*	Lat. (ϕ) ... *51° 05′ 00″*	R.O. *Lamp on Road.*
Page in Angle Book	Apprx. Long....	
Place ... ... *Beacon Hill.*	Bar. ... *28·95*	
Date ... ... *16.11.10.*	Ther. ... *35°.*	Mag. Bearing of R.O. *3° 42′ 46″*

$$\text{Tan } \frac{A}{2} = \sqrt{\sec s \sec (s-p) \sin (s-\phi) \sin (s-h)} \text{ and } s = \frac{h+\phi+p}{2}$$

A = Horizontal Angle between the Elevated Pole and the Sun or Star.
h = True Alt.

In this formula the latitude (ϕ) should be taken with the positive sign whether N, or S. and the Polar Distance (p) is then to be reckoned from the Elevated Pole.

AZIMUTH—*cont.*

ELEMENTS.

	′	″
Refraction due to Alt. =	*1*	*3·62*
Corr. for Bar ± =		*− 2·2*
Corr. for Ther. ± =		*+ 2·1*
Refraction ... ... =	*1*	*3·52*

Hor. Par. (page I., N.A.)
×
Cos. Alt. ... ... =

∴ Parallax in Alt.... =

COMPUTATION.

	°	′	″
From Angle Book / Mean Obsd. Alt. =	*42*	*32*	*19·8*
Corr. for Refraction... = −		*1*	*3·5*
Corr. for Semi-diam. ± =			
Corr. for Parallax ... = +			
True Alt. (*h*) ... =	*42*	*31*	*16·2*
Latitude (*φ*) ... =	*51*	*05*	*00*
Polar dist. (*p*) =	*81*	*22*	*04·3*
	2)*174*	*58*	*20·5*
	87	*29*	*10·3*

With Sun Obsns. only.

Omitted with M.T. Chron.

	h. m. s.
G.S.T. of G.M.N. (page II., N.A.) ... ... =	
Corr. for Long. at 9·86 sec. per hour (or Acceln. table 23 Raper) (+W. − E.) ... ... =	
L.S.T. of L.M.N. ... =	
Mean of Times (Angle Book) ... ... =	
∴ Sid. Int. from L.M.N. =	
Retardation or M.T. Equivalents (Table in N.A.) ... ... h. m. secs.	
∴ L.M.T. of Obsn. ... ... =	
(If M.T. Chron. used this is taken direct from Angle Book.)	
Long. E. (−) or W. (+) ± =	
G.M.T. of Obsn. ... ... =	
∴ Int. from G.M.N. in hrs.... =	
Decl. (δ) at G.M.N. (N. or S.) ± =	° ′ ″
Hourly Var. (page I., N.A.) × Int. from G.M.N. ± =	
Decl. (δ) of Sun. or * at time of Obsn. ... ... ± =	*8 37 55·7*
	90 00 00
∴ Polar dist. (*p*) ... ... =	*81 22 04·3*

Note.—"*p*" is reckoned from elevated pole, ∴ if *φ* and *δ* are of different signs "*δ*" will be negative and *p* = 90 + *δ*.

	°	′	″		
s =	*87*	*29*	*10·3*	Log sec =	*11·3579265*
s − p =	*06*	*07*	*6*	Log sec =	*10·0024808*
s − φ =	*36*	*24*	*10·3*	Log sin =	*9·7733907*
s − h =	*44*	*57*	*54·1*	Log sin =	*9·8492195*
					2)*7540·98301*

$$\text{Log tan}\,\frac{A}{2} = 10{\cdot}4915087$$

$$\frac{A}{2} = 72^\circ\ 07'\ 37''$$

2

$$A = 144^\circ\ 15'\ 14''$$

Angle between R.O. and ✱ (or Sun) (from Angle Book, $= 228^\circ\ 17'\ 38{\cdot}25''$

† ∴ Azimuth of R.O. from Elevated Pole $= 12^\circ\ 32'\ 52{\cdot}25''$

3. *Method* 2.—In this method the exact instant of each observation has to be recorded so that a knowledge of the error of the time-keeper is required. This involves a preliminary observation for time.

An R.O. is required as in Method I, and, in fact, in all methods for determining azimuth; the star is bisected with the vertical cross-hair of the telescope only, and the moment of this bisection recorded.

The elements of the astronomical triangle in this case are

(*a*) The Polar distance, p,
(*b*) The co-latitude, $90 - \phi$,
(*c*) The hour angle, t; deduced from the observed times.

The azimuth is given by the formula

$$\tan A = \tan t \cos M \operatorname{cosec} (+ M - \phi)$$

where

$$\tan M = \tan \delta \sec t.$$

In this method the horizontal limb of the theodolite only has to be read.

4. *Method* 3.—This method gives very accurate results and both observation and computation are simple.

The only objections to it are, that it cannot be used at or near the equator, and that, except when the observer is close to the poles, the number of stars suitable for observation is not very great, and those suitable may not be at elongation at times convenient for observation.

When the star is at elongation the parallactic angle is a right angle, and the elements of the astronomical triangle are two sides p, $90^\circ - \varphi$, and the parallactic angle 90°.

It is first necessary to select a suitable star and compute the time at which it elongates.

The hour angle of a star at elongation is obtained from the formula

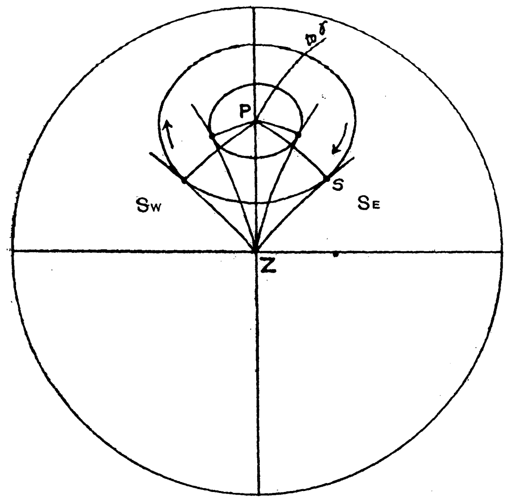
$$\cos t = \tan p, \tan \phi$$

From Fig. 49 it can be seen that t is the angle ZPS_W or ZPS_E, according as the star is at west or east elongation.

The R.A. of the star is the angle $SP\gamma$ so that

$$\left.\begin{array}{l}\text{At west elongation R.A.} + t\\ \text{At east elongation R.A.} - t\end{array}\right\} = \angle\, ZP\,\gamma$$

† Note.—Azimuth for Geodetic purposes should be measured from the South (by West). It is best ascertained from the above data by consideration of a figure showing the relative positions of Star, Pole and R.O.



Circumpolar Stars as appearing to observer above the Zenith.

FIG. 49.

The ∠ ZPγ is the interval which has elapsed since the transit of γ the First Point of Aries, and is therefore the L.S.T. of elongation.

Having worked out the L.S.T. of L.M.N. (Sect. 39 para. 9) the L.S.T. of elongation — L.S.T. of L.M.N. gives the number of *sidereal hours* between local mean noon and the time of elongation, converting these to M.T. hours gives the time of elongation in mean time.

5. It frequently happens that the only stars which elongate at convenient times are so small that it is difficult to see them except through the telescope, and that consequently it may be difficult either to lay the telescope on the star accurately enough to get it into the field of view or to be quite certain that, when a star is seen through the telescope, that it is the correct star.

It is advisable therefore to compute both the elvation and azimuth of the star beforehand, and set the telescope to the correct elevation and azimuth as accurately as possible (using the compass for azimuth if no better means is available) before commencing to observe.

Altitude is obtained from the formula*

$$\sin h = \sin \phi \sec p.$$

Azimuth from the formula*

$$\sin A = \sin p \sec \phi.$$

6. In selecting possible stars from the Nautical Almanac it is necessary to estimate as nearly as possible the value of the hour angle when stars of different declinations are at elongation.

Thus from Fig. 49 it can be seen that the nearer the star is to the pole, the nearer the hour angle approaches 90° or 6 hours.

* These formulæ are derived from "Napier's Circular Parts." See "Text Book of Topographical Surveying," page 200.

From the L.S.T. of L.M.N., work out the L.S.T. of the hour at which it is desired to observe.

This gives the angle ZP γ.

Estimate the angle ZPS in hours for stars of two or three declinations, and add or subtract from the L.S.T. of observation. This will give the R.A. of suitable stars. The Nautical Almanac should then be searched and the hour angles for one or two probable stars computed.

With a little experience it is generally possible to select a suitable star after the first computation of a "probable" star.

7. The time during which observations can be taken is limited to 10 minutes, 5 minutes before and 5 minutes after elongation. During this time the star moves very slowly in "azimuth" (the closer the star to the pole the slower will be the movement in azimuth).

The observations are—

F.L.	R.O.
F.L.	Star.
F.R.	Star.
F.R.	R.O.

Time should be taken to ensure that the observation is done within the prescribed limits, but it is usually easy to see from the motion of the star whether this is the case or not. Horizontal angles only need be read.

8. For artillery purposes, provided the instrument is in good adjustment, it is generally sufficient to observe on one face only and, having worked out the elevation, corrected to allow for refraction, the telescope is set at this angle, and laid on the star, which should be kept intersected with the vertical cross wire until it crosses the horizontal wire.

At this moment, and for a short time before and after, the star should appear to travel along the vertical wire.

The horizontal angle to the star as it crosses the horizontal cross wire should be read and recorded.

Example of Prediction of Circumpolar Star at Elongation.

Date, *Nov. 15th, 1922.*

It is desired to observe between 18·00 hours and 19·00 hours at a point Latitude 51° 12′ 01″ N. and Longitude 0 h. 07 m. 08 secs. W.

From Nautical Almanac.

On Nov. 15th G.S.T. of G.M.N. = *15 h. 35 m. 9·2 secs.*
In 07 min "γ" gains 7/60×9·86 secs. = *+1·2 secs.*

L.S.T of L.M.N. ... ... = *15 h. 35 m. 10·4 secs.*

L.S.T. of time of observation will be between 21 h. 35 m. and 22 h. 35 min.

The Hour Angle for stars of about 10° Polar distance is estimated in this Latitude to be about 5½ hours when the stars are at elongation.

Stars of about 80° declination and R.A. 22 hours + 5½ hours will be at E elongation.

Stars of about 80° declination and R.A. 22 hours − 5½ hours will be at W elongation. A search is made in the Nautical Almanac for stars of this declination and R.A. about 3½ or 16½ hours. It is found that ϵ Ursae Minoris (δ 82° 10′ 15·6″;

R.A. 16 h. 53 m. 38·6 s.) appears to fulfil the conditions. The elements for the star are computed thus :—

Time of elongation deduced from the Hour Angle "t" formula $\cos t = \tan \phi \tan p$.

$\tan \phi$ =	*10·0947371*	t =	*80° 09′ 11″*
$\tan p$ =	*9·1382985*	=	*5h. 20m. 37·secs.*
		RA =	*16 53 38·6**
$\cos t$ =	*9·2330356*		
		L.S.T. of Elongation =	*22 14 15·6*
		L.S.T. of L.M.N. =	*15 35 10·4*

Sidereal Interval from L.M.N. = *6h. 39m.05·2 secs.*

convert this to Mean Time Units—

For 6 sid. hours	subtract	*58·98 secs.*	
39 sid. mins.	do.	*6·39* do.	= − *1 m. 05·4 secs.*
05 sid. secs.	do.	*0·01 do.*	

M.T. Interval from L.M.N. = *6 h. 37 m. 59·8 secs.*

L.M.T. of Elongation = *18 h. 37 m. 59·8 secs.*

Compute Azimuth from $\sin A = \sec \phi \sin p$ and Altitude from $\sin h = \sin \phi \sec p$.

$\sec \phi$ = *10·2030096*	$\sin \phi$ = *9·8917275*
$\sin p$ = *9·1342313*	$\sec p$ = *10·0040670*
$\sin A$ = *9·3372409*	$\sin h$ = *9·8957945*
A = *12° 33′ 21″ W. of True North*	h = *51° 52′ 23″ (Star's Elevation)*
Grid N is E of True N. by *0° 25′ 00″*	
Star is W. of Grid N. ... *12° 58′ 21″*	Angle Book gives R.O. as being *176° 23′ 45″* measured clockwise, from star.
Grid Bearing of Star ... *347° 01′ 39″*	
Angle between star and R.O. *176° 23′ 45″*	
Grid Bearing of R.O. ... *163° 25′ 24″*	

44. Use of the Theodolite for Astronomical Observations.

1. The theodolite is set up and levelled as described in Chapter IV. If it has been set up by daylight preparatory to use by night its level should be checked before beginning to observe.

2. For observation on stars it is necessary to illuminate the cross wires of the theodolite. Some instruments are provided with a window in one trunnion into which a light can be shone and reflected by a small mirror on to the diaphragm.

A simpler and more effective method, which is applicable to any instrument, is to secure a strip of stiff paper 1 or 2 inches wide to the object end of the telescope and to bend it slightly over the object glass. The light of a torch or lamp is then shone on to the paper and reflected

* Note.—If the star had been at E elongation the L.S.T. of observation would have been found by subtracting the Hour Angle from the R.A. of the star.

down the telescope on to the cross hairs: the amount of light can then be varied by moving the lamp.

3. For observations on the sun it is necessary to protect the eye by a dark glass on the eyepiece of the telescope. If no dark glass is available it is possible to throw an image of the sun and cross wires on to a piece of white paper held a few inches away from the eyepiece: the focus of the eyepiece has to be altered to get a sharp image of the cross wires.

4. It is often difficult to bring a star into the field of view of the telescope, especially when the diagonal eyepiece has to be used.

In some cases the altitude of the star can be previously computed: this altitude is set on the vertical limb of the theodolite which is then traversed in azimuth until the star comes in the field.

Even if the altitude has not been previously computed, once the star has been found, the first reading of the altitude so read can be used for finding it again.

5. Whenever possible the star or sun should be allowed to make its own contact with the cross wire. When times are being booked, the observer says "Up" as the bi-section is made. Except when both horizontal and vertical angles are being read simultaneously, the exact intersection of the cross hairs should not be used.

6. In reading the instrument the bubble should always be read first: next the readings on the vertical limb: and lastly those on the horizontal limb.

7. *Duties of the Booker.*—A good booker not only relieves the observer of anxiety in connection with details of the stars to observe on, changing face, etc., but should also be on the alert to detect errors in reading or observing.

The booker gives the observer directions as to the star to be observed, the altitude to set on the theodolite to find the star, the face to use, and the observations required (vertical, horizontal, or both).

When times are being booked, the observer gives the booker, "stand by" a few seconds before obtaining the bi-section: the booker then begins to count half seconds by his timekeeper, *e.g.*, "seven, half, eight, half, nine, half, etc.," when he hears "up" he estimates the fraction of a second when it is given: with a little practice this can be done to a tenth of a second.

8. *Detail of booking.*—The detail that has to be booked for the various observations is given in the following specimen sheets of the angle book.

In observations to the sun, the booker should record the limb of the sun with which contact was made as it appears to the observer in the eyepiece.

A

Specimen Page of Angle Book.

Page 2.

Place, R.E. Camp, (near Chatham).
Date, 21st May, 1911.
Level Value, 5″
Height of Signal ——————
Height of Instrument——————

Observations for Time.
Name of Star, etc., Sun.
Chron. No. 42.
R.O. ——————

Bar, 29· 9″
Ther. 72° F.
Error of Chron. on L.M.T., Unknown.
Mag. Bearing ——————

Object.	Face.	Horizontal. A. °	A. ′	A. ″	B. ′	B. ″	Means of A and B. °	′	″	Vertical. C. °	C. ′	C. ″	D. ′	D. ″	Means of C and D. °	′	″	Level. E.	Level. O.	Chron. Times. h.	m.	s.
$\underline{\odot}$	L.	}								**216**	**27**	**59**	**28**	**13**	36	28	6	**13¼**	**14¼**	20	6	40·3
$\overline{\odot}$	R.	1st	deter	minat	ion					**323**	**35**	**31**	**35**	**47**	36	24	21	**12**	**16**	20	9	42·8
$\underline{\odot}$	R.									**322**	**42**	**23**	**42**	**40**	37	17	28·5	**12**	**16**	20	12	10·0
$\overline{\odot}$	L.	}								**217**	**13**	**52**	**14**	**5**	37	13	58·5	**13¼**	**14¼**	20	15	12·5
														Mean	36	50	58·5	50½	60½	20	10	56·4
												Level	corre	ction		+	6·25	$+\frac{10\times5''}{8}$				
										M	ean o	bserv	ed alt	itude	36	51	4·75	$=+6{\cdot}25''$				

Observer A.

Recorder Z

Eye-piece reverses vertically.

Note.—The entries in black type are those for which the observer is responsible; those in ordinary type are filled in by the booker.

SPECIMEN PAGE OF ANGLE-BOOK.

Place, R.E. Institute (Station No. 9).
Date, 7th June, 1911.
Level Value, 5″
Height of Signal ———
Height of Instrument ———

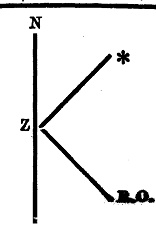
Observations for Azimuth.
Name of Star, etc. γ Cygni (East).
Chron. No. ———
R.O. Lamp at temporary beacon.

Bar 30·3′
Ther. 53° F,
Error of Chron. on L.M.T. (L.S.T.) ———
Mag. Bearing (not observed).

Object.	Face.	HORIZONTAL. A.			B.		Means of A and B			VERTICAL. C.			D.		Means of C and D.			LEVEL. E.	O.	CHRON. TIMES.		
		°	′	″	′	″	°	′	″	°	′	″	′	″	°	′	″			h.	m.	s.
R.O.	R.	**86**	**0**	**14**	**0**	**2**	86	0	8													
*	R.	**29**	**11**	**12**	**11**	**0**	29	11	6	**319**	**31**	**42**	**31**	**55**	40	28	11·5	**13**	**18**			
*	L.	**209**	**52**	**20**	**52**	**5**	209	52	12·5	**221**	**7**	**40**	**8**	**0**	41	7	50	**16**	**15**			
R.O.	L.	**266**	**0**	**5**	**59**	**50**	265	59	57·5													
					Horiz	ontal	Angle	betw	een				Me	an	40	48	0·75	29	33			
						R.O.	and	Star.				Level	corre	ction		+	5·00	+	$\frac{4\times5''}{4}$			
							56	49	2						40	48	5·75	=+	5″			
							56	7	45													
					Me	an	56	28	23·5													

Observer A.

Recorder Z.



NOTE.—The entries in black type are those for which the observer is responsible; those in ordinary type are filled in by the booker.

Specimen Page of Angle-Book.*

Page 5.

Place, R.E. Institute (Station No. 10).
Date, 29th June, 1911.
Level Value, 5″
Height of Signal ————
Height of Instrument ————

Observations for Azimuth.
Name of Star, etc., Sun.

Chron. No. 14
R.O., Spire on Tower of Naval Hospital.

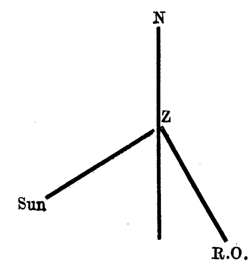
Bar ————
Ther. ————

Error of Chron. on L.M.T. 0 m. 32·1 s. slow.
Mag. Bearing (not observed).

Object.	Face.	Horizontal.									Vertical.								Level.		Chron. Times.		
		A.			B.		Means of A and B.			C.			D.		Means of C and D.			E.	O.			
		°	′	″	′	″	°	′	″	°	′	″	′	″	°	′	″			h.	m.	s.
R.O.	L.	**322**	**33**	**50**	**34**	**10**	322	34	0		Hori	zonta	l Ang	le bet	ween	R.O.	and S	un.		—	—	—
O	L.	**60**	**30**	**3**	**30**	**21**	60	30	12						97	56	12			4	3	51·0
O	R.	**241**	**28**	**51**	**29**	**9**	241	29	0						98	54	41			4	5	22·8
R.O.	R.	**142**	**34**	**10**	**34**	**28**	142	34	19					Mean	98	25	26·5		Mean	4	4	36·9

Observer A.

Recorder Z.



* This observation has not been computed; the computation form is given on p. 237.

Note.—The entries in black type are those for which the observer is responsible; those in ordinary type are filled in by the booker.

CHAPTER XII.

SURVEY APPLIED TO GUNNERY.

45. General Principles.

1. The object of survey methods and processes applied to gunnery in the field is to enable the line and range to a target to be predicted, and the gun to be laid in this line, with the utmost accuracy attainable in the time available for preparation. The accuracy with which the initial line of fire is laid out being of course of special importance when observation of fire for ranging or registration is for any reason impracticable.

2. The nature of the method to be used in any particular case will depend on the time available for applying it and the accuracy required. The latter, in its turn, depending on the nature of the target, and the precision with which it has been located, on the nature of the shoot, and of the gun carrying it out.

3. In dealing with small targets, such as hostile gun emplacements or machine guns, whose position has been accurately fixed, the degree of accuracy required will be conditioned by the performance of the gun. For example, in engaging such a target without observation it is desirable that the accuracy of prediction and laying should at least be sufficient to bring the target within the 100 per cent. zone and, preferably within the 50 per cent. zone of the gun.

4. If line and range can be predicted, and the gun laid, with an accuracy such that errors in these processes alone will not bring the target outside the 50 per cent. zone of the gun, then the average inaccuracies of fire due to laying will not exceed those due to irregularities in the performance of the gun itself or of its ammunition, and fire can be opened with immediate effect.

5. Owing to the shape of the 50 per cent. zone of all guns, errors in predicting or laying out line are much more destructive to the best performance of the piece than errors in range. It is therefore to the accurate prediction and laying out of *line* that artillery survey methods are mainly directed.

The width of the 50 per cent. zone depends on the nature of the gun and the range, but, if an average value is taken, it may be said that for the more accurate forms of predicted shooting, the combined errors of prediction and laying should not exceed an amount which will throw the shell 15 yards to one side or the other of the gun-target line.

6. If this is accepted as a standard, it is then necessary to consider what degree of accuracy is necessary in fixing the *positions* of the gun and aiming point to enable this standard to be attained.

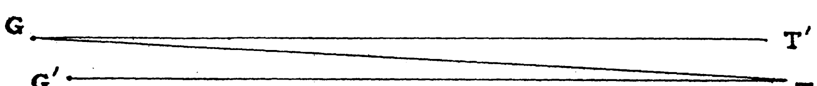


Fig. 50.

In Fig. 50 let G be a gun and T a target.

Suppose that the position of the gun has been fixed with a small error so that it is thought to be at G′ instead of at its true position G.

If the bearing of the target is deduced from the co-ordinates of gun and target, it will also be in error; being, in this case, too small by the amount of the angle GTG′.

If this bearing is laid out as a line from the true position G, the shell will be directed at a point T′ where TT′ = GG′.

In other words if the line as predicted could be laid out without error, the permissible error in fixing the position of the gun would be equal to the permissible error at the target, *i.e.*, 15 yards.

7. If, however, any error is made in laying out this line there will be another error at the target superimposed on the error in prediction, and this second error will be the product of the angular error *in laying* and the range.

If the laying is done direct off an aiming point it is necessary to consider not only how the error in fixing the gun position will affect the bearing of the target but also how it will affect the bearing to the A.P.

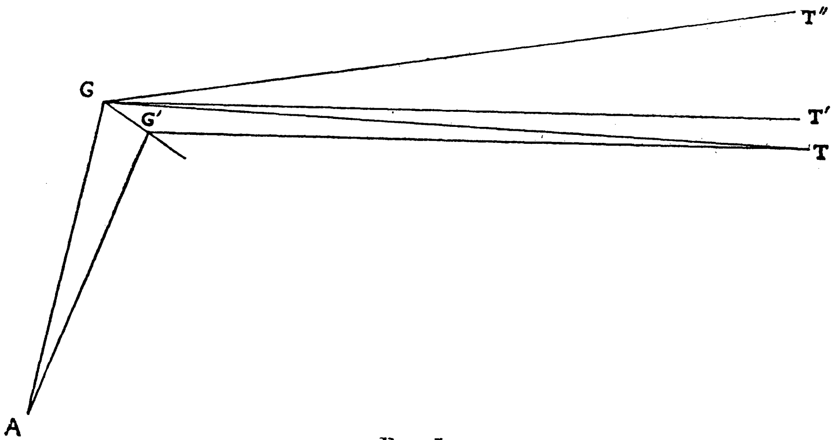


Fig. 51.

Thus in Fig. 51 if A is an aiming point, the correct angle which ought to be put on the dial sight is TGA. This angle is a combination of the bearings GT and GA. Owing, however, to G having been fixed wrongly at G′ both these bearings will be in error. GT by the angle GTG′ and GA by the angle GAG′. The line laid out will consequently be thrown off the target not only as far as T′ by the error GTG′ but by an additional amount T′T″ due to the error GAG′, the angle T′GT″ being equal to GAG′.

The magnitude of the angle GAG′ depends on the length of GG′ and the distance GA of the A.P. from the gun; for a given value of GG′ it varies inversely as GA, being greater when GA is less.

8. If the length GA were equal to the range GT, and the effect of the directions of the lines GA, GG′, GT on the errors in bearing is neglected (actually this effect is of course considerable, but, since the

true position of G is unknown, these directions cannot be determined, and the most unfavourable case must be allowed for) then it may be said that, of the total error at the target represented by TT″, one half T′T″ is due to the error in the bearing GA and one half TT′ is due to the error in the bearing GT (the first being an error in the *prediction* and the second an error in *laying*); and the length of GG′ should be one half of TT″.

9. If however, as is normally the case, GA is considerably less than the range GT, the proportion of the total error TT″, represented by T′ T″ due to errors in laying, increases. If GA is only one-quarter of the range, the error T′T″ will be four times and the total error TT″ will be five times as great as TT′ and the length of GG′ must not then exceed one-fifth of TT″; that is to say in such cases the error in fixing the gun should not exceed one-fifth of the permissible error at the target even if the aiming point is correctly fixed.

This standard of accuracy can only be attained by the employment of trigonometrical methods, which, owing to their slowness and the high standard of training and practice they require, are not well suited for *general* use.

10. It is desirable, if not absolutely necessary, to devise some method which, while attaining the requisite accuracy, allows of the gun position being fixed by the quickest and simplest possible method.

Graphic methods, such as plane table resection, if based on good and sufficient triangulation may be expected to give results within 10 yards of the truth, but cannot be relied on to give greater accuracy than this. If such methods are to be adopted for general use, errors in laying out, as opposed to predicting, the line, must not exceed an amount calculated to result in an error of 5 yards at the target at ordinary ranges.

For field guns this would imply that the angular error in laying should not exceed 2 or 3 minutes. (At 3,438 yards an error in laying of 1 minute will cause an error of 1 yard at the target. At 5,000 yards an error in laying of 3 minutes will cause an error of 4½ yards at the target. With larger guns firing at longer ranges the linear error corresponding to any particular angular error in laying out the line will be proportionately greater, but with such guns both the width of the 50 per cent. zone, and the effective radius of the burst are also proportionately greater, so that the permissible linear error at the target may be increased. An angular error of 3 minutes in laying out the line will therefore be quite accurate enough for the larger guns also.)

11. From the considerations set forth in para. 6 it is clear that such accuracy cannot be obtained if the position of the gun, as determined by graphic methods, enters into the calculations required for laying out the line, unless very distant aiming-points can always be used.

Distant aiming-points can, however, very seldom be used, and it follows, therefore, that the general employment of graphic methods of fixing gun positions is only permissible if the line can be laid out in some way not dependent on the position assigned to the gun.

This can be done without much difficulty if some point can be found or established, in the vicinity of the gun position, from which bearings to surrounding points *can* be determined within the required limits of accuracy.

If from such a point an accurate bearing to some other object is known, a director can be set up at the point and laid in any required line by simply laying off an angle from the object. The battery director or the pivot gun can then be laid parallel to the first director, as described in Artillery Training, Vol. II, Sect. 16.

12. The total preliminary work required for predicted shooting comprises three distinct operations, viz :—

(*a*) Fixation of the gun positions.
(*b*) Prediction of line, range and angle of sight.
(*c*) Laying out the line.

It is essential that each of these operations should be carried out by methods which, while giving the requisite accuracy, are simple enough for them to be carried out by a Battery Commander or his subordinates without *expert* assistance, which should be confined to preliminary work only, and which should be kept down to the minimum possible.

13. This *expert* preliminary work consists in the fixation of a few points, referred to in para. 11, from each of which accurate bearings to one or more surrounding objects have been determined. Such points are marked with " bearing pickets."

The same bearing picket, if sited properly, can be used by several batteries. Their number will therefore be much less than the number of the batteries using them and the objections to the use of trigonometrical methods for fixing them do not apply with the same force as when used for fixation of gun positions.

The accuracy with which line is *predicted* will depend only on the position assigned to the gun.

The accuracy with which it is *laid out* will depend on the accuracy with which the *bearings from the bearing picket to surrounding points* have been determined. It is not affected by errors in the positions assigned either to the gun or the A.P.

Normally bearing pickets will be established by Artillery Survey Companies and their positions determined trigonometrically.

Battery positions will be fixed by the batteries themselves. The two operations can thus proceed concurrently, and it may be expected that, in a new position, bearing pickets will be ready and available as soon as batteries are in a position to make use of them.

46. Fixation of Gun Positions.

1. Gun positions may be fixed by one or other of the following methods :—

(*a*) Plane tabling.
(*b*) Sub-base and director.
(*c*) Traversing.
(*d*) Measurement from map detail.

The best method to use in any particular circumstances will depend on the time available, the nature of the country, and the existing survey data.

2. When the survey data consists of triangulated points, plane tabling is generally most suitable, but much depends on the skill of the man using it.

In the hands of a well-trained and experienced man a plane table will give quick and accurate results. Its disadvantages are that it cannot be used at night or in the rain, and that both the speed and accuracy of the results obtained by it depend very much on the skill of the man using it. With an inexpert workman it may prove to be very slow and sometimes inaccurate.

3. Sub-base methods are especially applicable to positions in which a bearing picket or trig point can be seen from the battery, and is within 300 or 400 yards of it.

A sub-base of say 20 metres can be put out at the battery, one end of it being the dial sight of the pivot gun and the direction of the sub-base, laid out by the aid of the sight, at right angles to the line to the bearing picket or trig point.

The angle subtended at this point by the sub-base and the bearing of the dial sight can be measured at the same time with a director.

The co-ordinates of the pivot gun can be computed by bearing and distance from the bearing picket and the battery line can be laid out as described in Sect. 48.

The whole procedure can easily be reduced to a drill which can be carried out in less than half an hour. An example showing the form in which the computations should be carried out is given below :—

EXAMPLE OF BRIGADE SURVEY OFFICER'S NOTE BOOK.

Co-ordinates of point at which director is set up (measured from plane table).	E 457321	N 165176	
	A *Salisbury Spire.*	B *Cursus.*	C
E. Co-ordinate of Director ...	*457321*	*457321*	
E. Co-ordinate of each trig pt....	*458038*	*455651*	
Difference E	*717*	*1670*	
N. Co-ordinates of director ...	*165176*	*165176*	
N. Co-ordinates of each trig pt.	*149859·5*	*163140·3*	
Difference N	*15316·5*	*2035·7*	
Log difference E	*2·85552*	*3·22272*	
—Log difference N	*4·18516*	*3·30871*	
=Log tan bearing	*8·67036*	*9·91401*	
Bearing	*177° 19′*	*219° 22′*	

	A. Bty. ° ′	B. Bty. ° ′	C. Bty. ° ′	D. Bty. ° ′
Bearing to trig point	*177 19*	*177 19*	*177 19*	*177 19*
Angle to G.P.O.'s director ...	*45 11*	*53 45*	*62 51*	*63 14*
Bearing to G.P.O.'s director ...	*222 30*	*231 04*	*240 10*	*240 33*
Angle subtended by sub-base ...	*3 51*	*2 34*	*1 53*	*1 07*
Log cot subtended by sub-base...	*11·17201*	*11·34846*	*11·48304*	*11·71014*
Log sub-base (20 metre base) ...	*1·30103*	*1·30103*	*1·30103*	*1·30103*
Sum = Log distance	*2·47304*	*2·64949*	*2·78407*	*3·01117*
Log distance	*2·47304*	*2·64949*	*2·78407*	*3·01117*
Log sin bearing	*9·82968*	*9·89091*	*9·93826*	*9·93991*
Sum = Log difference E ...	*2·30272*	*2·54040*	*2·72233*	*2·95108*
Difference E	*200·8*	*347·1*	*527·6*	*893·5*
Log distance	*2·47304*	*2·64949*	*2·78407*	*3·01117*
Log cos bearing	*9·86763*	*9·79825*	*9·69677*	*9·69167*
Sum = Log difference N ...	*2·34067*	*2·44774*	*2·48084*	*2·70284*
Difference N	*219*	*280·4*	*302·6*	*504·5*
E. Co-ords. of Bde. S.O.'s director	*457321*	*457321*	*457321*	*457321*
Difference E	*200·8*	*347·1*	*527·6*	*893·5*
E. Co-ords. of G.P.O.'s director...	*457120·2*	*456973·9*	*456793·4*	*456427·5*
N. Co-ords. of Bde. S.O.'s director	*165176*	*165176*	*165176*	*165176*
Difference N	*219*	*280·4*	*302·6*	*504·5*
N. Co-ords. of G.P.O.'s director	*164957*	*164895·6*	*164873·4*	*164671·5*

This method is likely to be especially useful when a position has to be occupied after dark and fire has to be opened at once or at daylight next morning.

4. Traversing is most likely to be required in enclosed or wooded country, or when it is necessary to work at night and positions visible from a bearing picket or trig point cannot be occupied.

For rapid work or for traverses not exceeding half a mile in length a plane table may be used or the angles may be measured with a prismatic compass and the traverse plotted graphically.

For traverses of greater length angles should be measured with a director or theodolite, and the work computed.

Distances may be measured in a variety of ways. For short traverses up to a mile in length taping is generally possible and sufficiently rapid.

When the traverse is over ground which is free from obstacles a bicycle is a quick and sufficiently accurate method. Unless it is fitted with a cyclometer (which should be checked over a measured distance), the circumference of the front wheel should be measured, and a piece of white cloth attached to one of the spokes. The bicycle should be pushed along the line, and the number of times the white cloth passes the front fork counted, independently if possible by two men, so as to check the result.

Subtense or tacheometric methods of measuring distances are quick and convenient, if the special instruments and equipment required for them are available, or can be improvised.

5. During operations in extensive forests or in very enclosed country where triangulation and plane tabling has to be replaced by traverse work, bearing pickets will probably be replaced by lines of traverse running through the battery positions, and executed by survey companies.

The traverse stations should be suitably marked, and besides serving as bearing pickets will act as starting and closing points for any shorter traverses required (executed by the batteries themselves), for fixing the positions of their guns.

In all traverse work it is advisable to close the traverse either on the starting point or on another previously fixed point.

6. Fixing from map detail should be done if possible with the aid of instruments, either director, compass, or plane table by the ordinary methods of intersection, resection, or traverse, based on map points instead of trig points.

When no instruments are available an attempt should be made to make use of the methods of "prolongations" and "alignments"; that is to say, points should be searched for in the vicinity of the battery which are exactly in line with two map points in one direction and with two others in another direction as nearly as possible at right angles to the first.

If no such points can be found, resort must be had to pacing from the nearest map detail. Pacing is rarely sufficiently accurate for measurement of distances exceeding 200 or 300 yards.

7. At the same time that the *position* of the pivot gun is fixed its height should be determined.

When the plane table is used this can be done conveniently with an Indian clinometer.

When the gun is fixed by traverse or sub-base methods, vertical as well as horizontal angles must be taken. For this reason a compass is unsuitable except in flat country, for measurement of the angles of a traverse, and the present patterns of director much inferior to a theodolite.

47. Artillery Boards.

1. Having fixed the position of the pivot gun the next step is to work out the line and range to the target or zero point.

The best method of doing so is to compute them with the aid of logarithm tables (or a slide rule) from the formulæ given in Sect. 7.

Computation has the advantage that no time need be spent in mounting maps or in plotting points on a map or grid, and that no instruments are required for it. Its disadvantage is that a book of logarithm tables or a fairly large slide rule (a 20″-slide rule should be used) must be carried, and that arithmetical and other mistakes are likely to be made in it.

"Five-figure" logarithms are the most suitable, but four-figure tables are good enough for most purposes, and are included in the range tables for all guns. The occurrence of arithmetical mistakes must, however, be guarded against.

This can best be done by checking all computation by some graphic method.

2. For this reason it is necessary to provide all batteries with a method of determining line and range which can be made applicable to all circumstances, and which is at the same time quick, accurate, and simple.

3. Line and range can be determined graphically by plotting gun and target on a map, but this method has several disadvantages. The chief of these are :—

(1) Suitable maps may not be available.
(2) Gun and target may be on different sheets, and it would be very inconvenient to measure angles and distances without carefully cutting and joining the sheets together.
(3) The plotting and drawing of the lines must be done carefully, and a large protractor for measuring the bearings must be used.
(4) It is impossible to prevent expansion or contraction of the paper on which the map is printed, and this may cause considerable errors.

4. The use of a map for this purpose is both slow and unreliable, and it is necessary to make use of the special equipment known as an artillery board whose object is to provide a quick, simple, and reasonably accurate means of determining line and range graphically, as a check on computation.

5. An artillery board consists of a rectangular board made of three-ply wood generally similar to a plane table top. It is provided with a screw socket on its under side so that it can be attached to the service pattern plane-table tripod, and, if necessary, used as a plane table. It is provided also with the following instruments, &c., carried in clips on the under side.

(i) A right-angled template for marking out points on the grid.
(ii) A graduated ruler, pivoted at one end.
(iii) A special pivot for the ruler, provided with an arrangement for centring over a particular point.

(iv) A graduated arc.
(v) A small plotting scale.
(vi) A supply of drawing pins.

6. The artillery board should be prepared for use by mounting on it a piece of drawing paper in the manner described in Sect. **28**, and ruling up a grid on this.

This can be conveniently done by the aid of the template, which is punched with two lines of small holes at right angles to one another. The small holes are spaced at intervals of 1,000 metres on a scale of 1 : 20,000, so that they mark the corners of squares of the grid.

Having ruled in the grid in ink, the position of the pivot gun should be plotted in a convenient square near one edge of the board, and the S.W. corner of the square numbered in pencil to correspond with the co-ordinates of the pivot gun.

Once this square has been numbered, the co-ordinates of all other square corners can be written in, and any target plotted in its appropriate square.

The pivot should now be centred over the plotted position of the pivot gun and pinned to the board.

The pivoted ruler should then be placed on the pivot and laid exactly north and south, or east and west across the board.

This can be done conveniently by scaling off a distance on one of the grid lines equal to the appropriate co-ordinate of the pivot gun (*e.g.*, if it is desired to place the ruler exactly east and west across the board, the north co-ordinate of the pivot gun should be plotted on one of the grid lines 10,000 metres east or west of the pivot), and the edge of the ruler brought up against the point so found.

The ruler should be held in this position and the graduated arc placed on the board so that it butts against the recess provided in the ruler at the opposite end to the pivot, and so that the edge of the ruler is exactly over one of the larger graduation lines of the arc. In this position it should be pinned to the board by *one* pin, as nearly as possible in prolongation of the ruler.

The ruler should now be moved across the board, keeping it on its pivot, and the arc brought to bear evenly against the recess, in all positions of the ruler. When this position for the arc has been found, it should be pinned down finally by two or three pins distributed along its length.

The graduations on the arc should then be numbered in pencil on the paper beside it, and the board is ready for use.

To obtain the bearing and range of any target or other point the point is plotted on the grid and the pivotted ruler brought round against it. The bearing of the point can be read off on the arc, and its range on the graduations of the ruler.

If the plotting is carefully done the range obtained in this way should be within 20 yards and the bearing within 10 minutes of the computed values.

8. To prepare the board for use in a new position it is only necessary to remove the arc and pivot, and rub out the pencil figures on the grid.

Having re-plotted the position of the pivot gun the grid lines can be renumbered, and the pivot and arc replaced in their new positions in a few minutes.

9. The artillery board, once the paper has been mounted and the grid ruled in, provides a very rapid means of determining bearing and range, which is quite accurate enough for many purposes. It must be remembered, however, that mistakes are just as likely to occur in preparing and using it as in computing.

Although it may be used alone when time is very limited, it must be emphasized that such use should be exceptional, and that the true object of an artillery board is to provide a check on computation, and not merely to save a little time or labour.

48. Bearing Pickets.

1. The object of a bearing picket is to provide a simple and accurate method of laying out a line of fire.

A bearing picket marks the position of a point from which accurate bearings to one or more surrounding points have been determined. It consists of a short length of iron pipe sunk vertically into the ground so that the top of the pipe is flush with the surface. This pipe serves as the socket for an iron picket carrying a notice board on which is inscribed :—

(*a*) The name or number of the picket.
(*b*) The co-ordinates and height of the point.
(*c*) The grid bearings of two or more points, which can be seen from it.

2. Bearing pickets will normally be established by special R.A. Survey personnel, who will, when time permits, also put in at each bearing picket an auxiliary picket at a definite distance from the bearing picket itself. Unless the nature of the site prevents it, this auxiliary picket will be placed exactly grid north of the bearing picket, and at a distance of 100 yards or 100 metres from it, according as to whether the grid system in use is a yard or metre system.

This auxiliary picket is intended for use at night or in foggy weather when the surrounding points cannot be seen. Whenever it is used, great care should be taken in centring the director exactly over the picket. An error in centring of 1 inch will result, at this distance, in an error of about 1 minute in the bearing of one picket to the other.

3. The principle, described in Section 47, para. 11, on which the use of bearing pickets is based may be illustrated by the following example.

If the bearing to one of the points marked on the bearing picket is given as 295° 15′ and the battery line is 95° 12′ (Grid) the battery zero line is 295° 15′ — 95° 12′, *i.e.*, 200° 03′, left of the line to the point.

Since the director is graduated in degress and minutes right or left of the zero, and reads up to 180° only, 200° 03′ *Left* is equivalent to 360° — 200° 03′ or 159° 57′ *Right*.

If the degree scale plate of the director is set at this angle, laid on the point, usually called the reference object or simply R.O., and the base plate clamped in this position, the director sight when the degree scale plate is turned to zero will be laid on a bearing of 95° 12′. That is to say, it will be parallel to the battery zero line, which is then laid out as in the manner described in Artillery Training, Vol. II, Section 16.

4. It is to be noted that in this procedure :—

(*a*) The director at the bearing picket is not laid on the target or battery zero point but *parallel to the battery line.*

(*b*) No linear measurements of any kind are required.

(*c*) The accuracy of the method depends solely on the accuracy of the bearings from the bearing picket to the surrounding points, and the care and precision with which the necessary angles are measured on the director. Errors made in the determination of the gun position, though they will affect the accuracy of the prediction of the line, have no influence on the laying out of it.

5. When a triangulation of average accuracy is available as a basis, it may be expected that the bearings to surrounding points given on the B.P. notice board are within 1′ of the truth and that in measuring the angles with a No. 5 or No. 6 director errors should not exceed 2′ at most. The error made in laying out line should not in such cases ever exceed 3′.

6. Bearing pickets will normally be fixed by trigonometrical resection, computed by one of the methods described in Chapter X.

When the country is unsuitable for this method, *i.e.*, in flat wooded country, or in extensive forests, it may be necessary to fix their positions by traversing.

In such cases the " surrounding points " will usually be the traverse stations on each side. If these are within two or three hundred yards auxiliary pickets will be unnecessary.

In such country triangulation will probably be impossible, and the basis of all survey work will be traverses.

Every traverse station may be regarded and used as a bearing picket.

Traversing for this purpose should be done with a theodolite and steel tape or chain ; though for rapid work over short distances, subtense or tacheometric methods are very useful, particularly when the nature of the surface, *i.e.*, marshes or undergrowth, makes the use of a tape or chain difficult.

7. Any trigonometrical station or trig point may be used as a bearing picket, provided that the necessary data or information about it is available to enable the bearings to surrounding points to be determined.

8. The procedure in using a bearing picket is as follows :—
Method I. (Reference Fig. 52.)

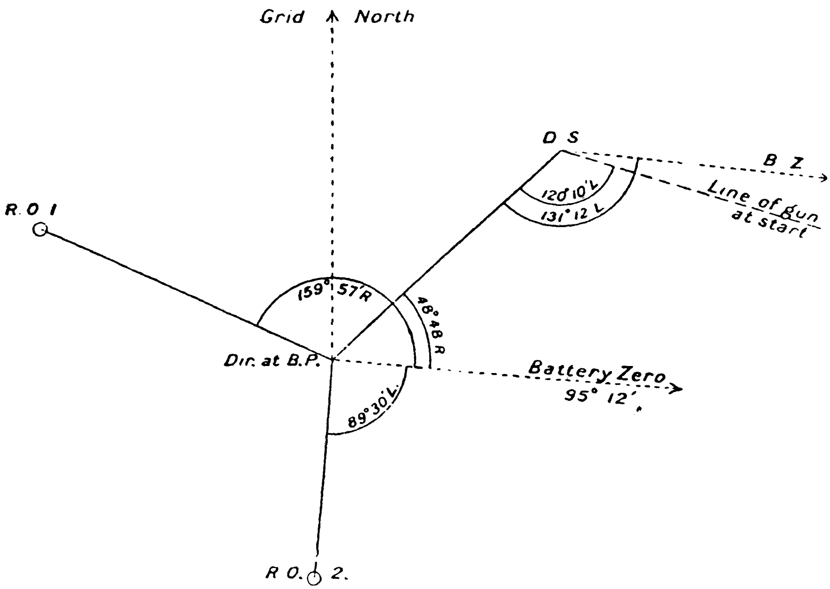


Fig. 52.

(*a*) Note the bearing to a reference object inscribed on the B.P. and find by subtraction the angle between this bearing and the battery line thus :—

Bearing of R.O.	295° 15′
Battery line	95° 12′
Difference	200° 03′ Left.

This being over 180° must be subtracted from 360° and is equivalent to 159° 57′ Right.

(*b*) Remove the bearing picket from its socket and set up a director vertically over the latter. Level the director and set the degree scale plate to read 159° 57′ Right. Release the base plate clamp and lay on the R.O.

Clamp the base plate and release the degree scale plate clamp.

The director is now laid with its zero on the line 95° 12′ (*i.e.*, the required line).

(*c*) Now proceed to lay the battery director parallel thus :— Turn the director sight at the bearing picket on to the battery director, and note the reading on the degree scale plate Record this ; suppose it to be 48° 47′ Right.

(*d*) In order to guard against mistakes in observing these angles or in reading the director (for example, with the No. 5 director it may be found that, in laying either on the R.O. or the battery director, the object has been intersected with one of the side graticuled wires instead of with the centre wire) repeat the above using a second reference object thus :—

Bearing of 2nd R.O.	184°	42′
Battery line	95°	12′
Difference	89°	30′ LEFT

Set the degree scale plate to this angle and lay on the 2nd R.O. Clamp the base plate and lay again on the battery director. Note the reading and record it. Suppose it is found to be

48° 49′ RIGHT.

Take the mean of this reading and that found in (*c*) above, *i.e.*,
48 ° 47′ RIGHT giving 48° 48′ RIGHT

If the disagreement between the readings exceeds 3 minutes, the observations should be repeated, using a third R.O. if such is available.

(*e*) Find the supplement of this angle 48° 48′ RIGHT, *i.e.*, 131° 12′ LEFT, and set the battery director (or dial sight) to this angle.

Lay the battery director by releasing the base plate clamp (or traverse the gun) until the sight is on the director at the bearing picket.

The battery director or gun is then in the required line.

9. *Method II* (*Reference Fig.* 53).—When line is being given direct to the pivot gun instead of to a battery director ; if the above procedure is adopted, it may happen that, when traversing the gun to bring the sights on to the director at the bearing picket, the dial sight will move sufficiently to necessitate the director at the bearing picket being laid on it a second time after the gun is approximately in the required line.

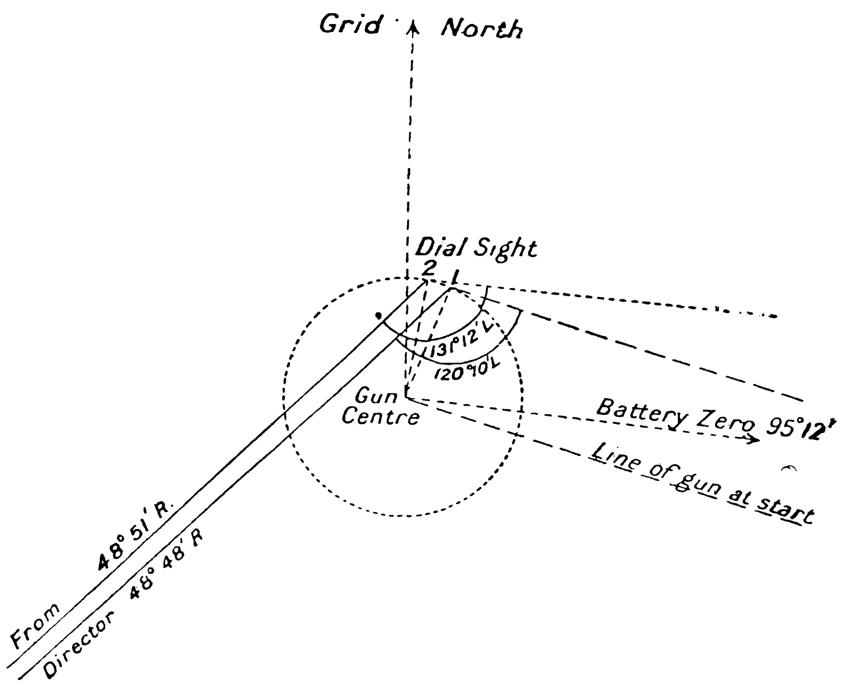


FIG. 53.

For example, if the dial sight describes a circle of 30 inches radius about the centre of motion of the gun, and the gun has to be traversed 11 degrees, the dial sight will move about 6 inches; and if the direction of this movement happens to be at right angles to the line joining it to the director at the bearing picket and the distance of the latter is 100 yards or less, the angle at the director will change nearly 6′.

10. This movement of the gun sight, which would involve repetition of the readings to the dial sight at the bearing picket, can be allowed for by a variation in the procedure after the angles to the dial sight have been observed on the bearing picket director. As follows:—

Having obtained the reading as above to the dial sight, viz., 48° 48′ RIGHT. *Without moving the trail* or traversing the gun pick up an aiming-point and note the reading to it on the dial sight.

Suppose this to be 5° 07′ RIGHT.

Again without moving the gun turn the sight on to the director at the bearing picket and note the reading, suppose it is found to be:— 120° 10′ LEFT.

Now take the supplement of the angle read at the bearing picket, viz., 48° 48′ RIGHT. This is 132° 12′ LEFT.

This angle represents the reading of the dial sight to the director at the bearing picket as it *should* be when the gun is laid in the required line. Actually the reading is found to be 120° 10′ LEFT.

The line of the gun is therefore 131° 12′ — 120° 10′ = 11° 02′ too much RIGHT. That is to say, the gun requires a traverse of 11° 02′ more LEFT to bring it into the required line.

This traverse can be given by setting the dial sight 11° 02′ more LEFT, that is to a reading of 5° 55′ LEFT (5° 07′ RIGHT — 11° 02′ more LEFT = 5° 55′ LEFT) and traversing the gun until the sight comes on to the A.P.

A few examples of the procedure using both methods are given below:—

	Ex. I.		Ex. II.		Ex. III.	
At bearing picket	°	′	°	′	°	′
Bearing R.O. 1	295	15	54 (+ 360°)	07	215	03
Battery zero line	95	12	89	26	127	37
Difference	200	03 L.	324	41 L.	87	26 L.
Set director on R.O. at	159	57 R.	35	19 R.	87	26 L.
Reading to dial sight	48	47 R.	96	32 L.	134	46 L.
Bearing R.O. 2	184	42	275	14	3 (+ 360)	32
Battery zero line	95	12	89	26	127	37
Difference	89	30 L.	185	48 L.	235	55 L.
Set director On R.O. 2 at	89	30 L.	174	12 R.	124	05 R.
Readings to dial sight	48	49 R.	96	35 L.	134	42 L.
Meaning of 1st and 2nd readings ...	48	48 R.	96	34 L.	134	44 L.
Supplement	131	12 L.	83	26 R.	45	16 R.

For *Method I* set these supplements on the battery director and lay on the director at the bearing picket.

For *Method II*, before moving the gun.

	Ex. I.		Ex. II.		Ex. III.	
	°	′	°	′	°	′
Dial sight when laid on director reads	129	10 L.	89	13 R.	32	47 R.
Correction	11	02 M.L.	5	47 M.L.	12	29 M.R.
Dial sight on aiming point reads ...	5	07 R.	12	17 L.	9	38 R.
Correction	11	02 M.L.	5	47 M.L.	12	29 M.R.
Setting for dial sight on aiming point	5	55 L.	18	04 L.	22	07 R.

Set the dial sight at these readings and traverse the gun until the A.P. comes on.

11. In both these methods it has been assumed that the dial sight can be seen from the director at the bearing picket. It may happen, however, that this cannot be done owing to the configuration of the ground, or intervening obstacles to vision, or because the dial sight is so near that the director cannot focus on it.

When this is the case a second director should be used to establish one or more subsidiary stations. Having selected a point from which both the director at the bearing picket and the dial sight can be seen, the second director is set up and laid on the battery zero line by Method I described in para 9.

The gun can then be laid parallel to this director by either method In setting the second director to the battery zero line there is of course only one R.O., namely, the director at the bearing picket, and especial care must be taken, by repetition of reading, to guard against blunders.

12. It is possible in this way to transfer the line from bearing picket to gun through several subsidiary stations. Thus, having set the director No. 2, director No. 1 can be moved to a third point and laid parallel in its new position to No. 2, which can then in its turn be moved to another position. If, however, more than one subsidiary station is required it is generally preferable to adopt one of the methods described in Sect. **49.**

Each subsidiary station should be marked by a picket driven into the ground, over which the director is afterwards carefully centred (since the displacement of only 1 inch is equivalent to an error of nearly 1 minute of arc at 100 yards).

Special case of railway gun.

13. The railway gun is traversed by running round a curve so that the dial sight, instead of moving a few inches, moves many yards, and the bearing to it from the bearing picket alters rapidly if the direction of the bearing picket is more or less at right angles to the section of the curve on which the gun is moving.

The difficulty can be got over in the manner shown in the following example :—

In Fig. 54—

Let B be the bearing picket.

C be centre of circular arc on which gun moves.

X be position of gun at first.
GXH be tangential to the curve at X.
BS be parallel to the required battery zero line.
XK be parallel to BS.

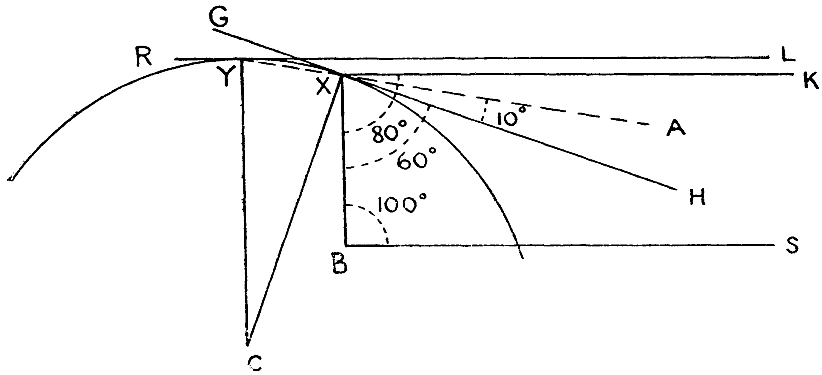


FIG. 54.

Suppose angle read on director at B = 100° R.
Then $K\hat{X}B = 80°$.
Let the dial sight at X read 60° L on the director at B, *i.e.*, $B\hat{X}H = 60°$.
Therefore the dial sight would have to be altered 20° more L.
Choose an aiming point A half this amount R, *i.e*, 10° R, *i.e.* A is on the bisector of $K\hat{X}H$

Let AX produced cut circle again in Y, and let RYL be a tangent to the curve at Y.

Put an angle equal to a combination of 20° L and 10° R, *i.e.*, 10° L on the dial sight and lay on A. This will bring the gun to Y. ∵ $L\hat{Y}A = G\hat{X}Y = A\hat{X}H = \frac{1}{2}K\hat{X}H = 10°$.

And because $L\hat{Y}A = \frac{1}{2}K\hat{X}H = K\hat{X}A$, YL is parallel to XK and therefore to BS. Hence the gun is laid in the desired line.

Now it will seldom be possible to choose A exactly in the bisector $K\hat{X}H$. Suppose in the above example we can select one at 11° 40′ R. Combining this with 20° L we obtain 8° 20′ L as the angle to put on the dial sight, and lay with on A. This will bring the gun not to Y, but to a point sufficiently close to Y, at which the final adjustment can be made on the internal traverse of the gun as in the case of a field piece.

49. Use of the Sun, Moon or Stars for Laying Out Line.

1. The sun, moon or stars are so distant from the earth that the lines to them from points on the earth several miles distant from each other are practically parallel. If the grid bearing of one of the heavenly bodies at any instant can be measured, the grid bearing to the same body from all surrounding points up to a distance of several miles at least will be the same at that instant.

2. The grid bearing of a heavenly body can easily be measured at a trig point or bearing picket or any point from which the bearing to some other visible point is accurately known. If a suitable signal can be made at this point to indicate the instant at which the bearing of the heavenly body is being observed, the angle between the same body and a reference point at any other point can be measured at the same instant, and the bearing of the reference point deduced.

3. It is unnecessary to know the exact positions of the observer or the reference point at any of the stations of observations.

4. This fact can be made use of for laying out lines of fire, and it is useful when the view from the gun is much restricted, or when it has not been possible to establish a bearing picket sufficiently near the battery, or when guns have to come into position at night without preparations for giving them line.

5. The sun can only be used when it is low or dulled by haze, or when smoked glasses are available to cut down the light.

Serious damage will be done to the eye by looking at a bright sun with a director or dial sight unprovided with a dark glass. There is also some risk of cracking the eyepiece.

The moon can be used whenever it is visible either by day or night, the "bright limb," *i.e.*, the fully illuminated circular edge, should be used for the observation. There is no difficulty in identifying which edge this is as, except when the moon is exactly full, it is always much more sharply defined when viewed through a telescope than the other edge.

Stars or planets may be used at any time of the night. When any choice is possible low stars which are moving slowly in azimuth should to selected for observation, but the most important consideration is be select stars about whose correct identification there can be no doubt.

6. Having selected the heavenly bodies which are to be used for observation, communication between the trig point and the points to which line has to be given must be arranged for, these latter points will be called "receiving" stations. The trig point will be referred to as the "sending" station.

The signal may be given by telephone, wireless or visual signal such as a rocket.

The procedure is as follows :—

At the sending station.—A director is set up, and made to read zero on grid north, by putting the bearing to one of the R.O.s on to the base plate, and laying to that R.O.

At the receiving station.—The director is set up, and made to read zero on a reference point, which may be some natural object, or for night work, a night aiming point marked with a lamp.

The approximate times of observation must be pre-arranged, and five minutes before these times the observers at all stations bring the telescopes of their instruments on to the star (or sun, etc., as the case may be).

Ten seconds before the observer at the sending station makes the observation a warning signal should be given and from this point all observers at the receiving stations should keep the star bisected by the central vertical cross wire of their instruments.

When the observer at the sending station makes his bisection of the star the signal is sent out. All observers stop traversing their instruments, and read and record the horizontal angles.

The procedure should be repeated at least once as a check.

The sending station then works out the grid bearings of the star, and sends them to receiving stations, who work out the bearings of their reference points thus :—

1st observation.

Sending station reads bearing of star 68° 42′.
Receiving station reads angle to star from R.O. as 22° 46′ R.
Grid bearing of R.O. is 45° 56′.

2nd observation.

Sending station reads bearing of star 67° 34′.
Receiving station reads angle to star from R.O. as 21° 36′.
Grid bearing of R.O. is 45° 58′.

Mean of two observations gives grid bearing of R.O. as 45° 57′. If the star is well selected it should not be more than 1 minute of arc in 4 or 5 seconds of time, so that observations sufficiently simultaneous are not difficult to obtain even with visual signals.

50. Application in Moving Warfare.

1. Bearing pickets will normally be established in convenient positions for the batteries which have to make use of them. It will hardly be possible, however, to establish them, even in the most favourable circumstances, sufficiently quickly for the use of batteries which have to advance and occupy new positions during the course of a battle.

2. In such cases the batteries have to fix their positions by rapid, and, if necessary, approximate methods; at the same time, the importance of accurate line is in no way diminished. If a good map is available it should be possible to determine the positions of the guns within an error of 20 yards, but it is by no means certain that an A.P. sufficiently distant to give an accurate bearing from such a roughly fixed point will be visible. Some map point should be searched for close to the gun from which a good view can be obtained and from which some really distant fixed point is visible. The bearing from this map or " auxiliary " point to the distant point should be calculated and the point used in exactly the same manner as a bearing picket. Assuming the position of the auxiliary point is not more than 20 yards in error with reference to the distant point (a reasonable assumption with a good map) and the distant point is ten miles away, the error in the initial bearing will not exceed 5 minutes of arc. A far more accurate result than can be expected if only a compass, or if the bearing from the gun to an aiming point even two or three thousand yards distant, is used.

3. With wireless sets the method given in Section **49** may be very useful in such cases, as the transmitting set can be located at some convenient point behind the line of departure from which accurate bearings can be determined. Fine weather, however, is essential, so that the sun or moon can be seen.

51. Use of the Compass.

1. The compass is not a very satisfactory instrument for giving accurate line for the following reasons :—

(*a*) The compass is liable to deflection by iron in its vicinity, which may be visible (guns, steel helmets, telegraph wires, etc.), or invisible (as magnetic ore, buried shells, etc.)

(*b*) The variation of the compass varies at different times of the day, though this diurnal variation can be to some extent allowed for by means of a table.

(*c*) The variation of the compass is occasionally affected by magnetic storms.

(*d*) The compass is liable to errors of construction, especially error of centering of the pivot in the divided circle.

The compass should therefore be regarded as a rapid, rather than an accurate, instrument for giving line, whose use, where accurate work is concerned, is mainly in checking other methods to prevent the occurrence of " gross " errors.

2. To see that a compass is in good working order lay it on a smooth horizontal table—say a plane table ; then with the finger against the vane projecting from the cover turn it smoothly about the centre.

If the compass card or needle does not remain steady but drags round after the case, then, either the point of suspension is damaged, or the needle, or the card is fouling something, probably the lever for lifting it off the pivot or else the check spring.

As a second test deflect the needle or card by bringing a piece of iron close to it, and see that it returns to rest in the same position when the iron is removed.

3. The compass gives the best results when used on a stand. A plane table or director stand, provided it contains no iron, may be used for this purpose.

Readings of the compass should be made by estimation to tenths of a degree.

4. The magnetic variation from grid north is generally shown in the margin of all military maps. This information is however not always to be relied on, and it is preferable to determine the variation of each individual compass on the ground at two, three, or more bearing pickets or trig stations.

Several independent determinations at different places are always desirable owing to the possibility of undetected iron in the vicinity of any single point. If accordant results are obtained at several different points errors from this cause are eliminated.

5. The deviation of the compass has at present a general diminution

of about 9′ per year. The diurnal variation is greater in summer than winter and varies from 5 to 10 minutes. It has its average value about 8 a.m. or 6 p.m. Between these hours the needle moves west to a maximum about 2 p.m., and then back to normal. After 6 p.m. it moves somewhat east, remains nearly constant during the night, and moves west again in the early morning to normal about 8 a.m.

These changes ranging over several tenths of a degree are hardly regular enough to make numerical correction desirable or possible. They can be at least partially avoided by doing compass work as much as possible in the early morning or late afternoon.

6. The following table shows approximately the safe distances from various objects made of iron likely to be met with on service :—

Heavy gun	60 yards.
Field gun	40 ,,
Telegraph wires	40 ,,
Barbed wire	10 ,,
Steel helmet	3 ,,
Cap badge, box respirator ...	½ yard.

The iron socket of a bearing picket will not affect a compass on a stand 3 or more feet above it. Steel spectacle frames (or rolled gold frames, which are steel made) must never be worn when observing with a compass.

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